PhODS MOCK 2022 PHYSICS COMPETITION

2022 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

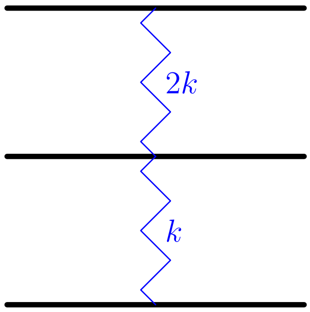
- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Test under standard conditions, meaning that you must complete the test in 75 minutes in one sitting.
- This test contains 25 multiple choice questions. **Your answer to each question must be marked on the Google Forms answer sheet that accompanies the test.** Proctored students will be given a separate form. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- Correct answers will be awarded one point; incorrect answers and leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- Join our discord to discuss the exam and join a large physics community!
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 1, 2022 EST.**

We acknowledge the following people for their contributions to this year's exam (in alphabetical order):

Abhiram Cherukupalli, Ashmit Dutta, Daniel Seungmin Lee, Pritesh Mehta, Azimuddin Sheikh

Special thanks to all testsolvers who were able to give helpful feedback to the problems.

1. A simple scale consists of two platforms of negligible mass fitted together by springs. When the scale is put on the ground, your weight W is measured by the deformation of the springs. Suppose you now fit the same number of springs on the upper platform of twice the stiffness of the lower springs. You then put another platform of negligible mass on the upper springs and then stand on the scale again to have the scale observe a weight W'. What is the ratio of the new observed weight to the original, W'/W_0?



(A) 1/3
(B) 3/2
(C) 2/3
(D) 3
(E) 2

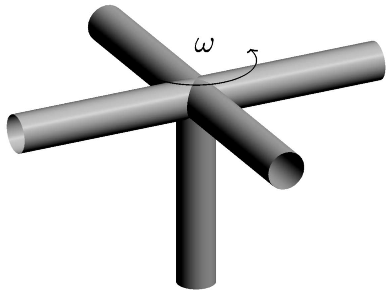
2. A small mass is put on the end of an ideal string of length $8a$ which is fastened to a vertex O on a small fixed cube of sidelength a. The mass-string system is extended to its maximum length and put parallel to one side of the square. The mass is then given a velocity v perpendicular to the string. In how much time will the mass hit the cube?

(A) $10\pi a/v$
(B) $14\pi a/v$
(C) $18\pi a/v$
(D) $20\pi a/v$
(E) $28\pi a/v$

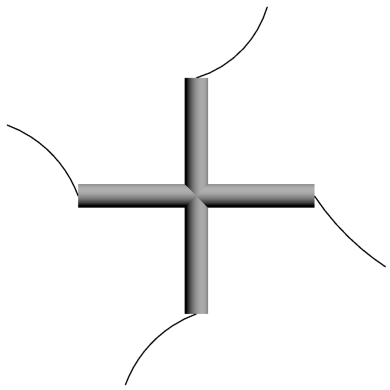
3. A sprinkler can be modelled as a hemisphere with N evenly divided holes, where water comes out at equal velocities. The maximum height that water reaches from the sprinkler is h. What is the maximum possible height achieved by the water when all the holes on the sprinkler, except one, are covered? Assume the initial flow rate of water that enters the hemisphere never changes with time.

(A) $N^{-1}h$
(B) h
(C) Nh
(D) N^2h
(E) N^3h

4. Consider a setup consisting of a four-way small pipe connected to a tube from below. The structure rotates at an angular velocity ω anti-clockwise and water flows upwards into the pipe at a constant rate. What would the pattern of water that flows out of the pipes look like from a ground observer watching from top?



(A)



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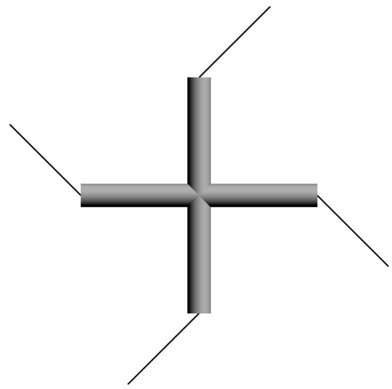

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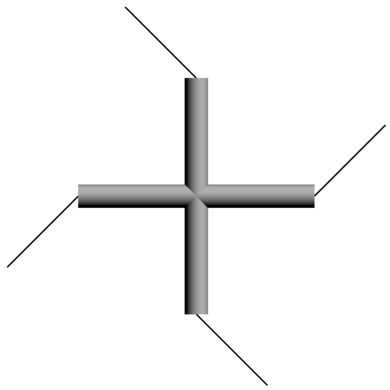

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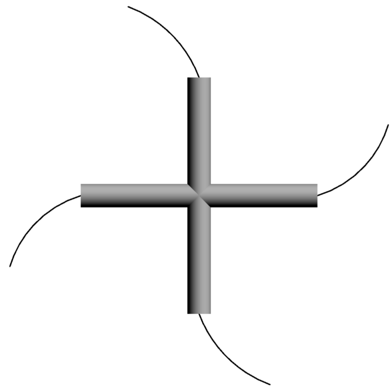

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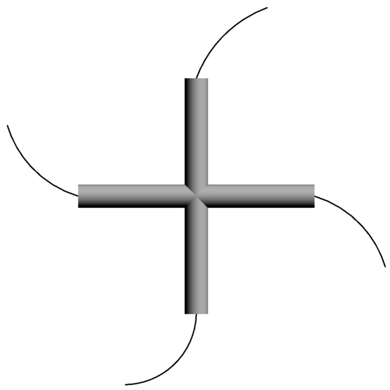
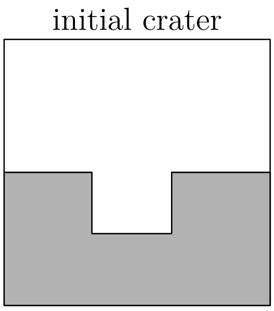
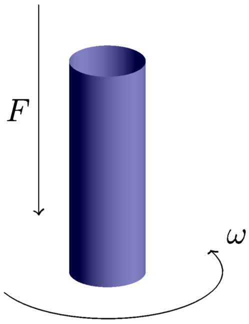


5. A meteor hits the moon and creates a crater within the ground. The crater can be modelled as a cubic hole of length L, created when the material originally within it is ejected out after the collision. Assume that energy is not lost during the collision. If a meteor of twice the mass of the initial meteor hits the moon at the same velocity, how much larger is the volume of the crater compared to the initial one? Assume everything on the moon are of the same density.



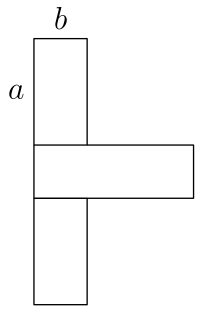
(A) 16
(B) 4
(C) 2
(D) $2^{3/4}$
(E) $2^{1/3}$

6. A thin hollow cylinder of radius $R = 10$ cm and thickness $\delta \ll R$, is situated vertically on a plane with kinetic and static coefficient of friction 0.3. A force $F = 10$ N is applied normal to the plane, and the cylinder is made to rotate at a constant angular velocity $\omega = 2$ rad/s. How much heat is dissipated due to friction after a time t?



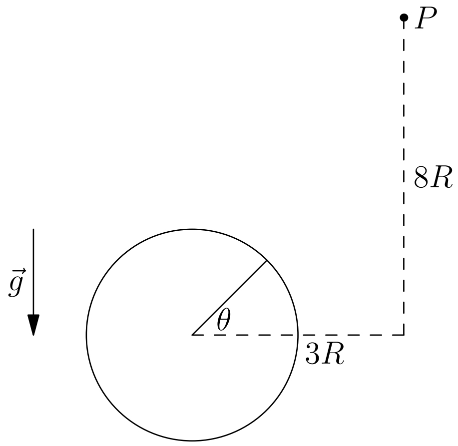
(A) $0.6t$
(B) $0.8t$
(C) $1.2t$
(D) $0.9t$
(E) $1.8t$

7. A tower is glued together from three identical wooden blocks of edge lengths a and b as in the figure below. What is the largest ratio a/b such that the tower would still be stable? The figure is not necessarily stable.



(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

8. In a vertical plane, where gravity acts downwards, a circle of radius R is put at the origin, and a bead is placed at point $P(3R, 8R)$. If the angle to the horizontal measured anti-clockwise is θ, at which of the following angle can a point Q on the circle be placed such that, if a rod through which the bead can freely slide without friction is put between points P and Q, the time of travel will be minimal? The speed of the bead is zero when released.



(A) $\theta = 60°$
(B) $\theta = 53°$
(C) $\theta = 45°$
(D) $\theta = 37°$
(E) $\theta = 30°$

9. A small hole is made on the side of a vessel of flat base and unknown shape which is placed on a flat table. The hole is covered by a piece of tape which can withstand a maximum of 6 N of force before breaking apart. Water is added to the vessel very slowly. Find the magnitude of force exerted by fluid on the vessel just after the tape breaks.

   (A) 3 N
   (B) 6 N
   (C) 9 N
   (D) 12 N
   (E) 18 N

10. A thin disk of mass M and radius R is rotating about its central axis at an angular velocity ω_0. The disk has a very small hole made at a distance $R/3$ from its center. The hole is suddenly covered by a rod and the rod stays at rest afterwards. What angular velocity does the disk starts to rotate ? Assume disk is lying on a frictionless surface

   (A) $2\omega_0/3$
   (B) $5\omega_0/8$
   (C) $7\omega_0/13$
   (D) $6\omega_0/7$
   (E) $9\omega_0/11$

11. A beam balance is used to measure the mass of a solid which has a very low density 0.5 g/cm^3. This solid is placed in the left-hand pan and metal weights of very high density are placed on the right-hand side. What is the percent error of the mass of the solid if the measurement is carried in a dense gas which has a density of 10 kg/m^3?

   (A) 1%
   (B) 1.2%
   (C) 1.5%
   (D) 2%
   (E) 2.3%

12. A weightless ladder of length $L = 2$ m rests at an angle of $\theta = 30°$ to the horizontal between a wall and a ground, both of coefficient of friction $\mu = 0.2$. A construction worker gets on top of the ladder. What is the maximum possible height the construction worker can be on the ladder for it to not slip?

   (A) 0.15 m
   (B) 0.25 m
   (C) 0.35 m
   (D) 0.55 m
   (E) 0.85 m

13. A dam is a large wall that holds water. The dam is of width L and water is at a depth H near the dam. How much compressive force acts from the water on the dam? The density of water is ρ.

   (A) $\rho g L^2 H$
   (B) $\rho g H^2 L$
   (C) $\frac{1}{2}\rho g H^2 L$
   (D) $\frac{1}{2}\rho g L^2 H$
   (E) $\frac{3}{2}\rho g \sqrt{L^3 H}$

14. A box of mass m contains a spring attached to its ceiling with an oscillating mass m attached below it. It is released from a height H in which the mass continues to oscillate in the frame of the box. When hitting the ground all the residual kinetic energy of the box is dissipated by heat. What is the minimum height the box must be released from to bounce back after the collision? The box is tall enough such that the inner mass does not hit its walls.

(A) $mg/3k$

(B) mg/k

(C) $mg/2k$

(D) $3mg/2k$

(E) $2mg/k$

15. A miniature carousel can be modelled as a disk of mass 2 kg and radius 1 m which rotates at an angular velocity 10 rad/s about a fixed axis through its center. Sand is then poured uniformly across the carousel such that the mass per unit area per unit time at which the sand hits the carousel is 0.2 $\text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$. Sand sticks to the surface of the carousel on contact. What is the angular acceleration in radians per second squared of the carousel immediately after the sand is poured?

(A) $\frac{3}{10}\pi$

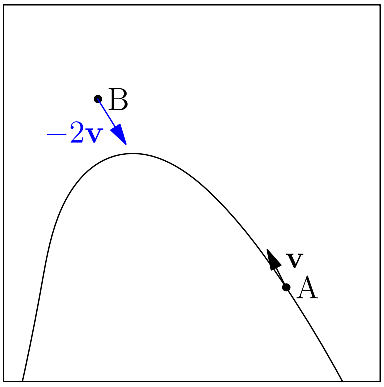
(B) $\frac{2}{9}\pi$

(C) $\frac{8}{9}\pi$

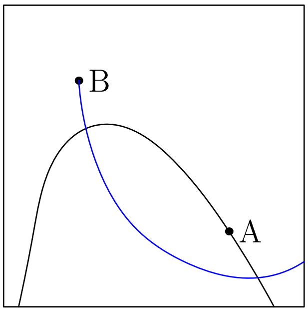
(D) π

(E) 2π

16. In an isolated system, two point masses A and B of $2m$ and m respectively interact with each other. If at the moment in this picture, the velocity of A is $\mathbf{v}$ and the velocity of B is $-2\mathbf{v}$, which of the following graphs would represent the trajectory of B?



(A)



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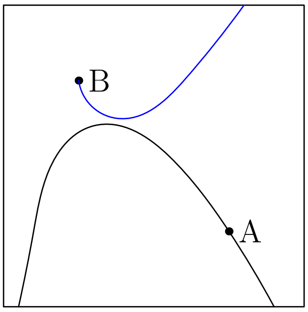

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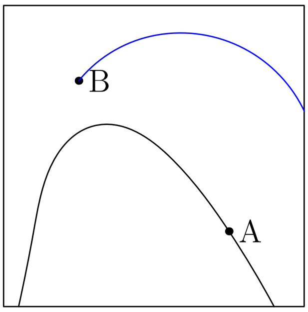

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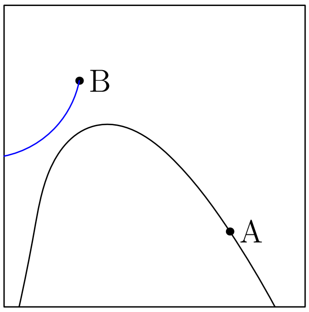

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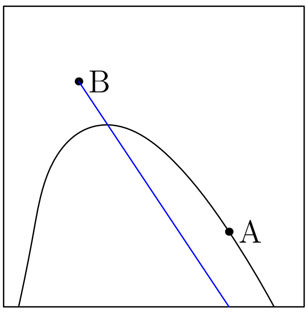
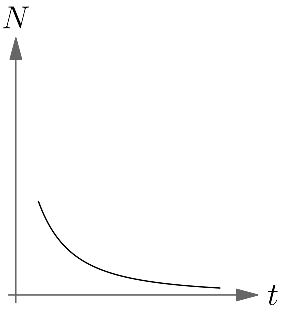


17. In a firework display, a firework rises in the air vertically and then explodes into fragments with equivalent velocity at all directions at a certain height. Which of the following graphs depicts the number of fragments N hitting the ground with respect to time t?

(A)



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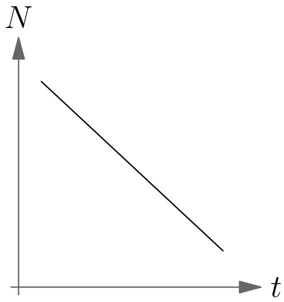

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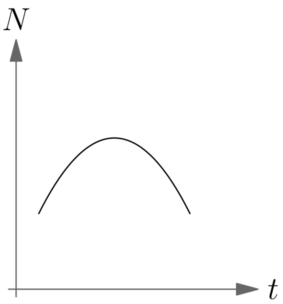

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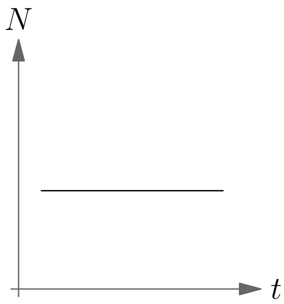

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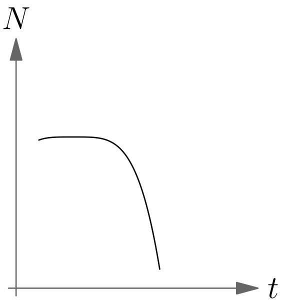
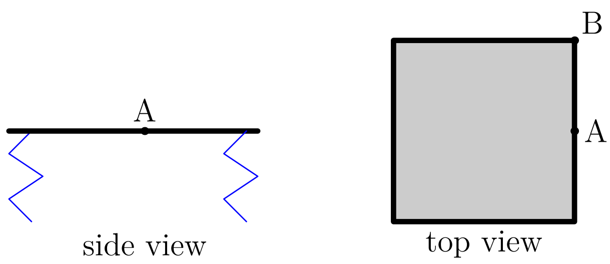


18. A massless spring with constant $k = 100$ N/m which is attached to the ceiling of a 10 m room. A small block of mass 1 kg is attached to the end of the spring. An identical spring is attached under the block. This pattern is repeated, adding blocks and springs until the contraption touches the floor. If the length of a relaxed spring is 50 cm, how many springs are needed?

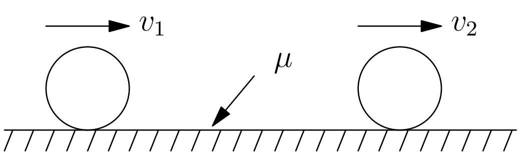
   (A) 6
   (B) 9
   (C) 10
   (D) 11
   (E) 20

19. A stretchy table consists of a square of mass m and sidelength a with fixed legs made of springs of constant k. The table is gently pushed down at the middle of one of the edges - point A in the figure - and the resulting angular frequency is measured to be ω_0. What is the angular frequency of the table when it is put at rest and now pushed down at its corner, B? Assume the center of mass does not move at all before and after being compressed.



   (A) $\sqrt{2}\omega_0$
   (B) $\sqrt{3}\omega_0$
   (C) ω_0
   (D) $2\omega_0$
   (E) $3\omega_0$

20. Two small rings of the same mass are put on a horizontal rough surface (μ=1) as shown in the figure. Ring A is rolling with velocity $v_1 = 10$ m/s towards ring B and ring B is rolling with a velocity $v_2 = 4$ m/s away from ring A. Find the final separation between the two rings after all possible collisions. Assume all collisions to be perfectly elastic and there is no friction interaction between the rings.



   (A) 0.9 m
   (B) 1.8 m
   (C) 3.6 m
   (D) 4.9 m
   (E) 9.8 m

The following information applies to problems 21-23.
A spaceship of mass m encounters cosmic dust of uniform density and definite edges. The dust has no initial kinetic energy and sticks uniformly to the spaceship and does not fall off after.

21. In deep space, the spaceship passes through the dust cloud. After passing through, its mass has increased by 2%. If the time taken to pass through the same area with no dust is T_0, then what is the time taken by the spaceship through the dust cloud?

(A) $0.99T_0$
(B) $1.01T_0$
(C) $1.02T_0$
(D) $1.03T_0$
(E) $1.05T_0$

22. The spaceship now flies at a *constant* speed v away from the surface of the Earth, which has a radius R and mass M. The spaceship encounters cosmic dust immediately from take-off which sticks at a constant mass rate of μ no matter what velocity the spaceship travels at. When the spaceship is at a distance $2R$ from the center of the Earth. Find the value of velocity v that minimizes the amount of thrust exerted by the spaceship at this position.
Note: The thrust of a spaceship is also known as the force needed to propel itself.

(A) $\sqrt{\frac{GM}{4R}}$
(B) $\sqrt{\frac{GM}{R}}$
(C) $\sqrt{\frac{2GM}{R}}$
(D) 0
(E) Very Large (infinite)

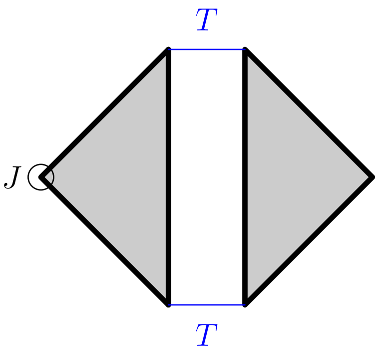
23. In this part, suppose the spaceship takes the form of a sphere of uniform density ρ_0. It remains stationary within a non-rotating dust cloud which has a density $\rho \ll \rho_0$, rotating at an angular velocity ω_0. When dust particles hit the spaceship, they stick to it. The spaceship grows in mass in a way such that its density remains ρ_0. How much time does it take for the spaceship to start rotating at an angular velocity $n\omega_0$ where $0 \ll n < 1$?

(A) $\sqrt{\frac{3\pi(n^{-\frac{3}{5}}-1)}{16G\rho}}$
(B) $\sqrt{\frac{3\pi(n^{-\frac{3}{5}}-1)\rho_0}{16G\rho^2}}$
(C) $\sqrt{\frac{3\pi(n^{-\frac{3}{5}}-1)\rho}{64G\rho_0^2}}$
(D) $\sqrt{\frac{3\pi(n^{-\frac{3}{5}}-1)}{64G\rho_0}}$
(E) $\sqrt{\frac{3\pi(n^{-\frac{3}{5}}-1)}{64G\rho}}$

24. A person is standing at the British Royal Observatory in Greenwich England, which is at a latitude of 53°N. The velocity of the sun relative to person that is measured by him at 6AM is closet to which of the following? The radius of the earth is 6400km and the Earth-Sun distance is 1.5×10^{11} m. Ignore effects occurring when speeds are near the speed of light.

(A) 1.1×10^7 m/s
(B) 3×10^4 m/s
(C) 4.7×10^2 m/s
(D) 1×10^2 m/s
(E) 1.28 m/s

25. A square of sidelength a and mass $2m$ is cut across its diagonal which is situated in the z-axis. Two ideal light rods of negligible length are put in between the two corners that were cut. A tangential impulse $J\hat{x}$ (into the page) is applied to the outermost corner of the square. What is the tension developed in the rods?



(A) $\frac{9(2-\sqrt{2})J^2}{ma}$

(B) $\frac{27(2-\sqrt{2})J^2}{ma}$

(C) $\frac{18(2-\sqrt{2})J^2}{ma}$

(D) $\frac{3J^2}{8ma}$

(E) $\frac{10\sqrt{2}J^2}{ma}$