AoPS Mock Physics Competition

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2020 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU START YOUR TIMER

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Test under standard conditions, meaning that you must complete the test in 75 minutes in one sitting.
- This test contains 25 multiple choice questions. **Your answer to each question must be marked on the Google Forms answer sheet that accompanies the test.** Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- Correct answers will be awarded one point; incorrect answers and leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after January 28, 2020.**

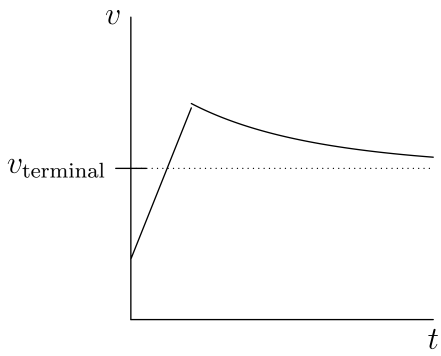
We acknowledge the following people for their contributions to this year's exam (in alphabetical order):

Ashmit Dutta, Aryansh Shrivastava, Viraj Jayam, QiLin Xue

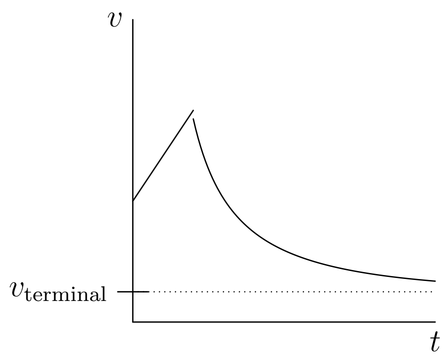
1. The 2020 US Physics Team wanted to avoid the hassle of airport baggage, so they built a rocket to take their belongings to Lithuania. The rocket is able to reach space, though they realized they forgot to attach parachutes just as it launched from Washington D.C. Let $t = 0$ be the time when the rocket is beginning to descend.

   Which of the following graphs best represents the speed of the rocket? You may assume the initial speed is near orbital speeds.

   (A)

   (B)

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   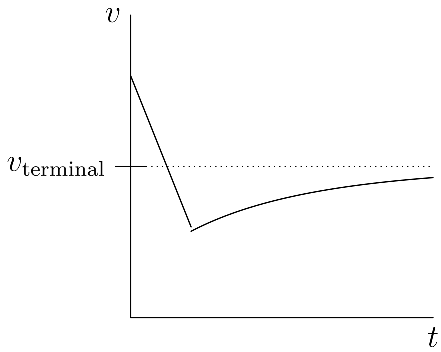

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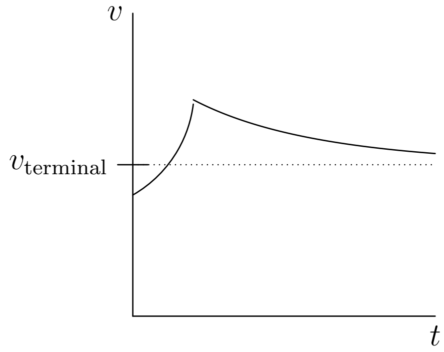
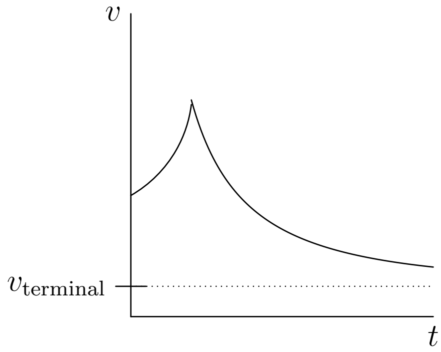
   

   (E)

2. Felix takes a ride in a Ferris wheel. The carriage records his "weight" in kg once he sits inside. Given that the Ferris wheel has a constant radius r, with what angular velocity should the Ferris wheel move such that Felix's weight at the top of the Ferris wheel is recorded as 0 kg on the scale?

   (A) gr

   (B) $\sqrt{\frac{g}{r}}$

   (C) $\frac{2g}{r}$

   (D) $\frac{1}{2}gr^2$

   (E) It is impossible for Felix to have a weight of 0 kg, as he exerts a constant force of mg down on the scale wherever he goes.

3. It is 2050, and a robot is attempting to measure the total length of a classroom. To do so, it picks up a meter stick and starts measuring from the first end. Due to the length of the meter stick, it must make a mark after it has measured about a meter, and then move the end of the meter stick to that mark to make the next measurement. Assume that the robot's measurement is exactly perfect and that the meter stick is poorly calibrated to an order that it suspects of 5%. What is the uncertainty in this measurement if the robot finds the classroom to be about 14.54 m wide?

   (A) 0.27 m

   (B) 0.72 m

   (C) 0.83 m

   (D) 1.03 m

   (E) 1.34 m

4. Jan throws a bowling ball with radius 0.6 m so that at some point in time it moves 2 m/s leftward while rotating 4 rad/s counterclockwise about its center of mass. At this point in time, what is the direction and type of friction acting on the ball at its point of contact with the floor?

   (A) Leftward static friction

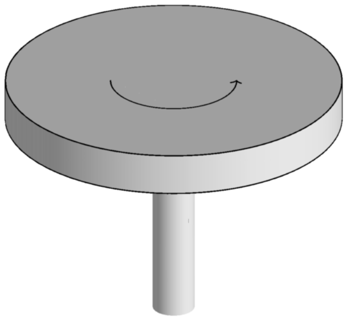
   (B) Leftward kinetic friction

   (C) Rightward static friction

   (D) Rightward kinetic friction

   (E) Friction does not act

5. A toy electric top is comprised of two parts: a plastic disk of mass m_1 and radius r, and a thin metal rod of mass m_2 and length ℓ attached to the bottom side of the disk perpendicular at its center. A small generator encased inside the top allows it to rotate with an average power P about the rod from time $t = 0$ to $t = t_0$. The top does not precess. At time $t = t_0$, the angular momentum of the top about the rod is



(A) $\sqrt{Pt_0(m_1r^2 + m_2\ell^2)}$.

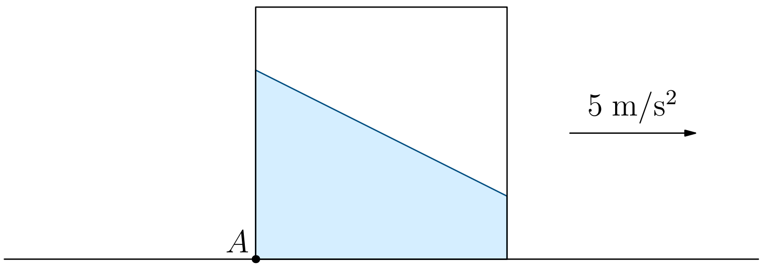
(B) $\sqrt{Pt_0\left(m_1r^2 + \frac{2}{3}m_2\ell^2\right)}$.

(C) $\sqrt{Pt_0\left(m_1r^2 + \frac{1}{6}m_2\ell^2\right)}$.

(D) $r\sqrt{m_1Pt_0}$.

(E) $\ell\sqrt{m_2Pt_0}$.

6. An aquarium in the shape of a cube with edges of length 2 m is half-filled with water of density 1000 kg/m^3 and sits on a frictionless surface. What is the pressure at the bottom left corner of the aquarium (denoted by point A) if the aquarium moves in the positive horizontal direction with acceleration 5 m/s^2.



(A) 1400 Pa

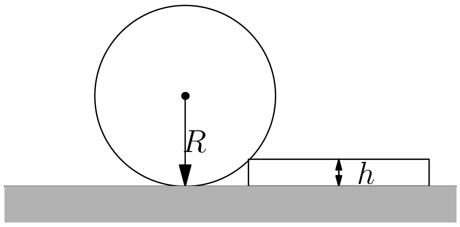
(B) 6100 Pa

(C) 9900 Pa

(D) 12000 Pa

(E) 15000 Pa

7. A cylinder has a radius R and weight G. It rolls without slipping towards the step as shown below. For the cylinder-step system, which of the following quantities are *not* conserved during the collision?



(A) Linear Momentum

(B) Angular Momentum

(C) Energy

(D) All of the above are conserved.

(E) At least two of the above are not conserved.

8. Alice and Bob each hold an end of a spring of spring constant k and very small mass δm. Alice pulls the left end of the spring leftward with a force of magnitude F_A, while Bob pulls the right end rightward with a force of magnitude F_B, where $F_A > F_B$. What happens to the spring?

(A) It is stretched by a length of $\frac{F_A}{k}$; its center of mass moves leftward with acceleration $\frac{F_A - F_B}{\delta m}$.

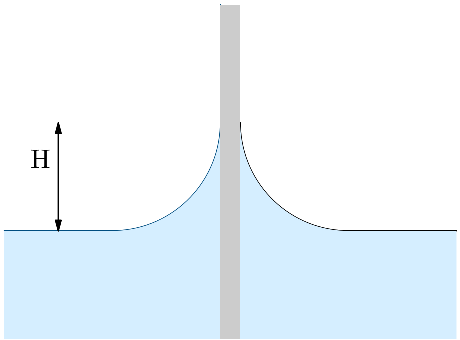
(B) It is stretched by a length of $\frac{F_B}{k}$; its center of mass moves leftward with acceleration $\frac{F_A - F_B}{\delta m}$.

(C) It is stretched by a length of $\frac{F_A + F_B}{2k}$; its center of mass does not move.

(D) It is stretched by a length of $\frac{F_A + F_B}{k}$; its center of mass moves leftward with acceleration $\frac{F_A - F_B}{\delta m}$.

(E) It is stretched by a length of $\frac{F_A - F_B}{k}$; its center of mass moves leftward with acceleration $\frac{F_A + F_B}{2\delta m}$.

9. A smooth vertical wall has been thoroughly wetted by water as shown in the figure below. The water clings to the wall due to surface tension, and therefore the water level increases by a height H. Now, the temperature of the surroundings increases, in turn making the density of the water decrease by 3% and the surface tension increase by 0.8%. Approximately, by what percent does the height of the water increase/decrease?



(A) 2% lower

(B) 3% lower

(C) 1% higher

(D) 2% higher

(E) 3% higher

10. An unstretchable string of length ℓ is connected to a lead sphere and hanged to the top of a cylindrical container a distance r from the center. The container rotates about its axis with an angular velocity ω which makes the pendulum create an angle θ to the horizontal. Now let the container fill up with water and yet once again rotate about its axis with an angular velocity ω. Which of the following must be true?

(A) The string rotates to an angle less than θ and has the same tension before.

(B) The string rotates to the same angle θ but has lower tension tension than before.

(C) The string rotates to the same angle θ and has equivalent tension as before.

(D) The string rotates to an angle greater than θ and has a lower tension than before.

(E) The string rotates to an angle greater than θ and has higher tension than before.

11. You are holding a baseball bat of mass m and length ℓ at its handle on the bottom, which is a distance d above the ground, in vertical position, assuming ideal conditions. You swing the bat through an angle θ at an angular acceleration α clockwise and then release it. The bat hits the ground with total energy

(A) $\frac{1}{3}m\ell^2\alpha\theta + mgd$.

(B) $\frac{1}{12}m\ell^2\alpha\theta + mg\,(d + \ell\cos\theta)$.

(C) $\frac{1}{3}m\ell^2\alpha\theta + mg\left(d + \frac{\ell}{2}\cos\theta\right)$.

(D) $m\ell^2\alpha\theta + mg\left(d + \frac{\ell}{2}\right)$.

(E) unknown, as there is not enough information.

12. The Little Prince is on a new planet with a grandfather clock calibrated on Earth. This new solar system is an exact copy of our solar system except all distances have been doubled. For example, the distance from the planet to the star will be doubled, and the radius will be doubled as well, among other lengths. The density stays the same. How many Earth years will the Little Prince measure to be one year on this new planet?

(A) $1/2$ Earth years

(B) $1/\sqrt{2}$ Earth years

(C) $\sqrt{2}$ Earth years

(D) 2 Earth years

(E) $2\sqrt{2}$ Earth years

13. Bronze is an alloy of copper and tin. In a certain batch of bronze, the copper used has a density of 9.32 ± 0.14 g/cm^3. The tin has a density of 6.56 ± 0.81 g/cm^3. The alloy is $78.00 \pm 0.02\%$ copper by volume. If all of the uncertainties are independent, what is the uncertainty in the density of the resulting bronze?

(A) 0.09 g/cm^3

(B) 0.12 g/cm^3

(C) 0.17 g/cm^3

(D) 0.21 g/cm^3

(E) 0.28 g/cm^3

14. A rope of uniform density λ moves under its own momentum at a constant velocity v. It approaches an incline at angle θ and moves all the way up until half of its length is on the incline. If the total length of the rope is L, then how much time elapses from the moment the rope begins to go up the incline until it stops momentarily? Assume the rope remains taut throughout the entire process.

(A) $\frac{\pi}{4}\left(\frac{v}{g\sin\theta}\right)^{1/3}$

(B) $\frac{v}{g\sin\theta}$

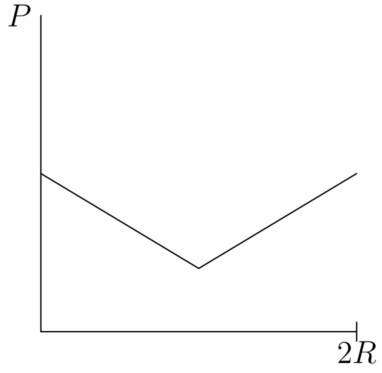
(C) $\frac{vg\sin\theta}{2L^2}$

(D) $\frac{\pi}{2}\sqrt{\frac{L}{g\sin\theta}}$

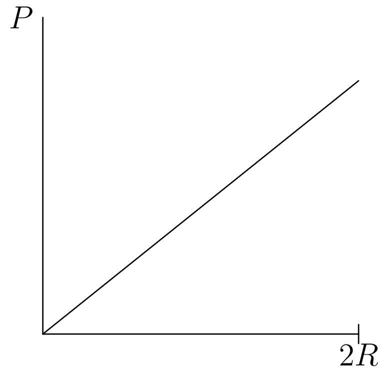
(E) $2\pi\sqrt{\frac{L}{g\sin\theta}}$

15. In deep space, a large cylindrical spaceship with radius R is rotating with an angular velocity ω. A drone takes off from the ground and in the frame of the spacecraft, flies directly upwards with a constant velocity to the other side a distance $2R$ away. In the frame of the rotating spaceship, which of the following shows the power P of the drone as a function of distance?

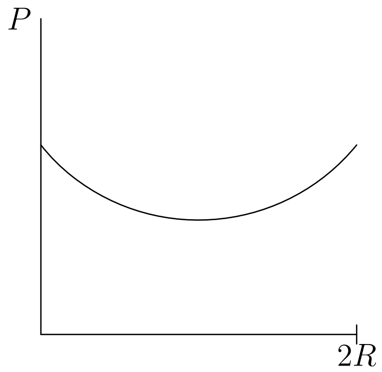
(A)



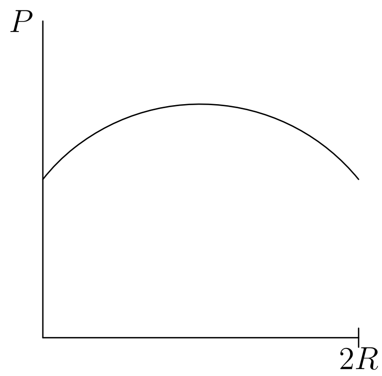
(B)



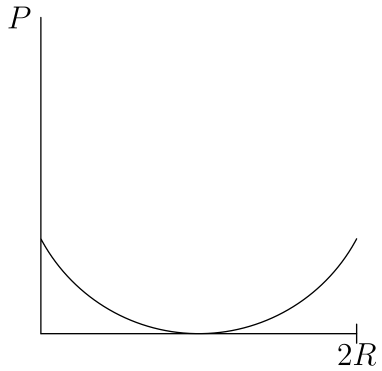
(C)



(D)



(E)



16. In xyz space, all coordinates are given in meters. A rectangle with uniform density is constructed with vertices at coordinates $(0, 0, 0)$, $(5, 0, 0)$, $(5, 4, 0)$, and $(0, 4, 0)$. Next, the quarter circle bounded by the arc between $(2, 0, 0)$ and $(0, 2, 0)$ centered at $(0, 0, 0)$ is cut out from the rectangle. If the remaining figure has mass 3 kg, what is its moment of inertia about the line passing through $(5, 4, 0)$ and parallel to the z-axis?

(A) 10 $\text{kg} \cdot \text{m}^2$

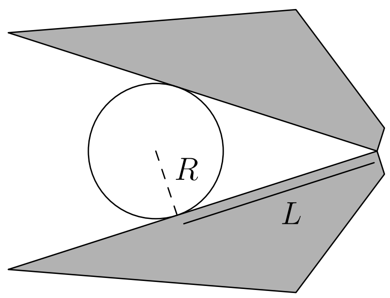
(B) 20 $\text{kg} \cdot \text{m}^2$

(C) 30 $\text{kg} \cdot \text{m}^2$

(D) 40 $\text{kg} \cdot \text{m}^2$

(E) 50 $\text{kg} \cdot \text{m}^2$

17. A clamp is used to pick up a gumball with mass m and radius R. The contact point is a distance L from the pivot. Then the clamp is held parallel to the page. The following diagram is a top-down view.



Given that the coefficient of friction is μ, what is the minimum normal force each arm needs to exert on the gumball?

(A) $\dfrac{mg}{\sqrt{\mu^2 - 1}}$

(B) $\dfrac{mg}{\sqrt{\mu^2 + (L/R)^2}}$

(C) $\dfrac{mg}{2\sqrt{\mu^2 - (L/R)^2}}$

(D) $\dfrac{mg}{2\sqrt{\mu^2 - (R/L)^2}}$

(E) The force of friction will increase as the normal force is increased, so it can withstand an infinite normal force.

18. A block of mass m is attached to a rope of initial length L, initial cross-sectional area A, and Young's modulus E. The rope stretches so that the volume stays constant. What is the stretch ΔL necessary in order to maintain static equilibrium when hung vertically?

    Note: the Young's modulus E is defined such that, for any force F, $F = AE\frac{\Delta L}{L}$.

    (A) $\dfrac{mgL}{AE}$

    (B) $\dfrac{mgL}{AE - mg}$

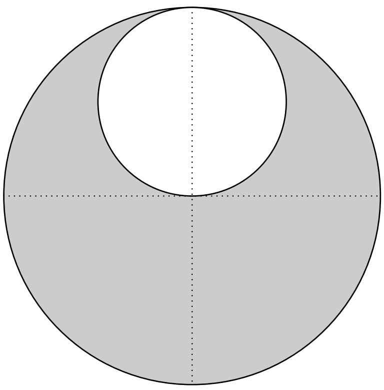
    (C) $\dfrac{mgL}{AE - 2mg}$

    (D) $\dfrac{2mgL}{AE - mg}$

    (E) $\dfrac{2mgL}{AE - 2mg}$

Use the following information for problems 19 and 20.

A hole of radius $R/2$ is cut out of a uniformly solid disk of radius R and mass M, which originally stands still on ground.



19. The object is slightly pushed so that it oscillates with respect to time. Find the period of oscillations given that there is sufficient friction to prevent the disk from slipping.

    (A) $2\pi\sqrt{\dfrac{20R}{67g}}$

    (B) $2\pi\sqrt{\dfrac{80R}{87g}}$

    (C) $2\pi\sqrt{\dfrac{24R}{20g}}$

    (D) $2\pi\sqrt{\dfrac{5R}{3g}}$

    (E) $2\pi\sqrt{\dfrac{29R}{10g}}$

20. A horizontal force is delivered to the top of the contraption in a very short period of time. What is the minimum impulse that needs to be delivered to make the disk rotate one full revolution? Assume it rolls without slipping.

(A) $M\sqrt{\frac{1}{12}gR}$

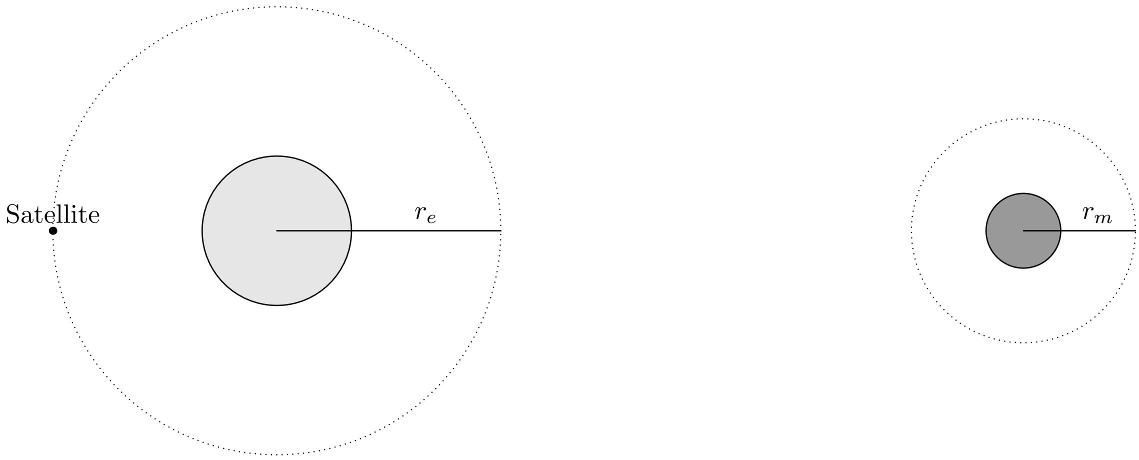
(B) $M\sqrt{\frac{64}{13}gR}$

(C) $M\sqrt{\frac{13}{256}gR}$

(D) $M\sqrt{\frac{27}{13}gR}$

(E) $M\sqrt{\frac{36}{13}gR}$

21. NASA wishes to bring a small rocket of mass m to the moon. First, it starts off in a circular orbit at a semi-major axis r_e around Earth. The engines fire in a negligible amount of time and bring the rocket to the moon, a distance d away. You may assume that $d \gg r_e$. Once the rocket reaches the moon, the engines fire once again and bring the rocket into a circular orbit at a semi-major axis r_m. The mass of the Earth and the moon are M_e and M_m respectively.



What is the total impulse the engines must impart on the rocket?

(A) $m\left(\sqrt{2}-1\right)\left(\sqrt{\frac{GM_e}{r_e}}+\sqrt{\frac{GM_m}{r_m}}\right)$

(B) $m\left(\sqrt{2}-1\right)\left(\sqrt{\frac{GM_e}{r_e}}\right)+m\sqrt{\frac{GM_m}{r_m}}$

(C) $m\left(\sqrt{\frac{GM_e}{r_e}}+\sqrt{\frac{GM_m}{r_m}}\right)$

(D) $m\left(\sqrt{\frac{2GM_e}{r_e}}+\sqrt{\frac{2GM_m}{r_m}}\right)$

(E) $m\left(\sqrt{\frac{GM_e}{r_e}}+\sqrt{\frac{2GM_m}{r_m}}\right)$

22. Two glass balls of equal mass are at rest on an infinite track. There is a distance d between the two balls. At time $t = 0$, Ball 1 is given a constant force F to the right. After every collision, the force F on the first ball is multiplied by 2. If all collisions are elastic, after a large number of collisions, what is the sum of the distances ball 1 travels between subsequent collisions?

(A) $10d$

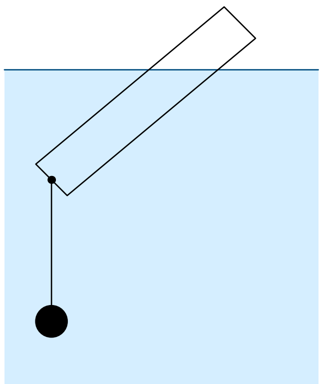
(B) $\frac{11}{2}d$

(C) $8d$

(D) $\frac{29}{3}d$

(E) Much greater than any of the above

23. A thin homogeneous cylindrical float is made out of a light substance with a density 0.5 g/cm^3. A lead sinker of density 204.53 g/cm^3 is tied with fishing line to the bottom of the float. Let the mass of the float be m, and the mass of the sinker be M. What conditions must the ratio of the masses of the sinker and the float satisfy for the float to rest vertically in the water? (Neglect the forces of surface tension. The density of water is 1 g/cm^3).



(A) $0.34 \leq \frac{M}{m} \leq 1.34$

(B) $0.41 \leq \frac{M}{m} \leq 1$

(C) $0.49 \leq \frac{M}{m} \leq 0.76$

(D) $0.57 \leq \frac{M}{m} \leq 1.53$

(E) $1.21 \leq \frac{M}{m} 1.73$

24. For all positive integers m, a block of mass $0.5^m \pm 0.2^m$ kg is placed at position m on the number line. The leftmost block is given velocity 1 ± 0.25 m/s rightward, and all collisions thereafter are inelastic. After all collisions occur, what will be the speed of the rightmost block, up to Gaussian uncertainty?

(A) 0.5 ± 0.26 m/s

(B) 0.5 ± 0.36 m/s

(C) 0.5 ± 0.47 m/s

(D) 0.5 ± 0.76 m/s

(E) 0.5 ± 1.22 m/s

25. A cylindrical rod of length $\ell = 0.5$ m is held such that the bottom is a distance h above the surface of the water. The density of the rod is a fifth of that of water. At $t = 0$, the rod is dropped from rest and is just about able to be submerged completely before rising. How long does it take for the rod to reach its lowest point? Ignore effects from splashing and drag.

(A) $\sqrt{\frac{2\ell}{g}} + \pi\sqrt{\frac{\ell}{5g}}$

(B) $\sqrt{\frac{3\ell}{g}} + \pi\sqrt{\frac{\ell}{5g}}$

(C) $\sqrt{\frac{2\ell}{g}} + \pi\sqrt{\frac{3\ell}{g}}$

(D) $\sqrt{\frac{2\ell}{g}} + \frac{\pi}{2}\sqrt{\frac{3\ell}{g}}$

(E) $\sqrt{\frac{3\ell}{g}} + \frac{\pi}{2}\sqrt{\frac{\ell}{5g}}$