PhODS MOCK 2021 PHYSICS COMPETITION

2021 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Test under standard conditions, meaning that you must complete the test in 75 minutes in one sitting.
- This test contains 25 multiple choice questions. **Your answer to each question must be marked on the Google Forms answer sheet that accompanies the test.** Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- Correct answers will be awarded one point; incorrect answers and leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after Feburary 1, 2021.**

We acknowledge the following people for their contributions to this year's exam (in alphabetical order):

Harsh Deep, Victor Dumbrava, Ashmit Dutta, Proelectro, Aarjav Jain, QiLin Xue, Daniel Yang

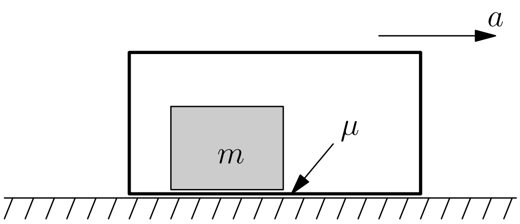
1. A baseball player hits a baseball with a force F causing the ball to fly a distance d_0. If he hits the baseball again with twice the initial force $2F$ under the same identical conditions as before (i.e. the contact time between the bat and the ball remains the same), then what is the ratio of the new distance traveled d to the original distance travelled d_0?

(A) $\sqrt{2}$
(B) 2
(C) $2^{3/2}$
(D) 4
(E) 8

2. A giant lilypad can be thought of as a large, uniform, circular cylinder of density 0.6 g/cm^3, thickness 4 mm, and radius 2 m. The lilypad is then placed in the middle of a large lake filled with water of density of 1000 kg/m^3. If a child is placed in the center of the lilypad, what is the maximum possible mass of the child (to the nearest 10 kg) such that the lilypad doesn't sink in the water?

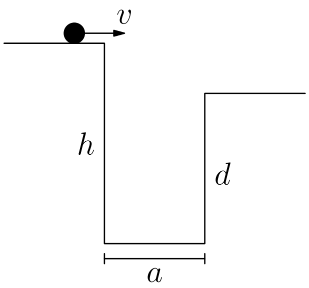
(A) 20 kg
(B) 30 kg
(C) 40 kg
(D) 60 kg
(E) 80 kg

3. A block of mass $m = 10$ kg is placed in a cart of mass 20 kg, which is kept on a frictionless table as shown in the image below. The coefficient of friction between the cart and the block is $\mu = 0.3$. Initially, both objects are rest, and at time $t = 0$, the cart is suddenly pulled by some external force with constant acceleration $a = 5$ m/s^2. Find the maximum non-impulsive force applied by the cart on the block when the block is at rest with respect to the cart. The answer choice may not be exact.



(A) 30 N
(B) 50 N
(C) 100 N
(D) 110 N
(E) 140 N

4. A ball flies off a small rectangular ditch with a height h (on the left side) and a height d (on the right side) with a width a as shown below. How much velocity does the ball need such that it lands on the other side of the ditch?



(A) $v > a\sqrt{\frac{g}{4(h-d)}}$

(B) $v > a\sqrt{\frac{g}{2(h-d)}}$

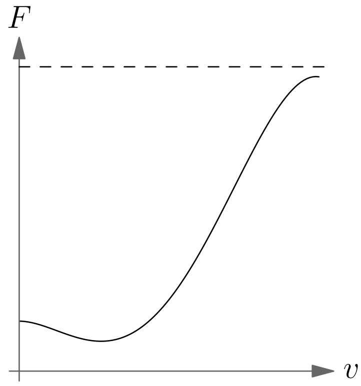
(C) $v > a\sqrt{\frac{g}{h-d}}$

(D) $v > a\sqrt{\frac{2g}{h-d}}$

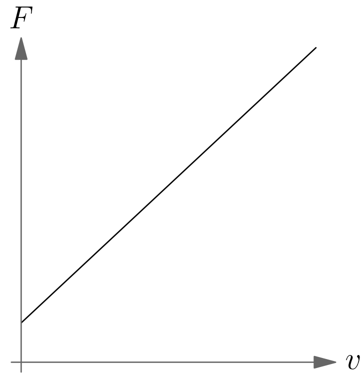
(E) $v > a\sqrt{\frac{4g}{h-d}}$

5. A runner is on a track. When the gun is fired to start the race, he begins running with his speed increasing over time. His speed is proportional to the frequency of his legs that propel him forward. Which of the graphs below show the amount force exhibited by his legs as he runs faster?

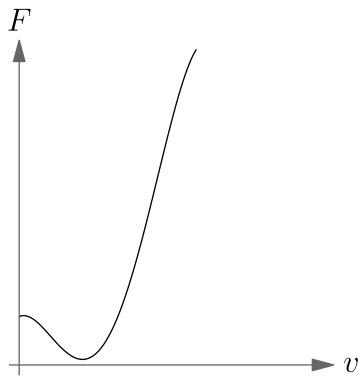
(A)



(B)



(C)



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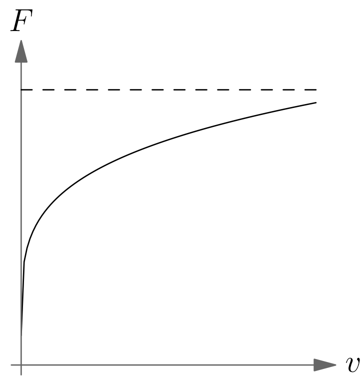
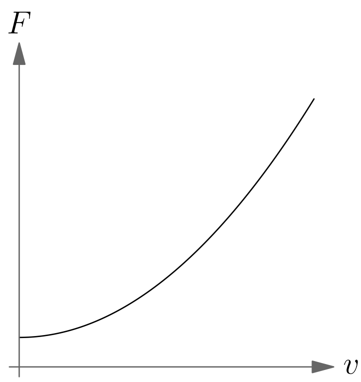


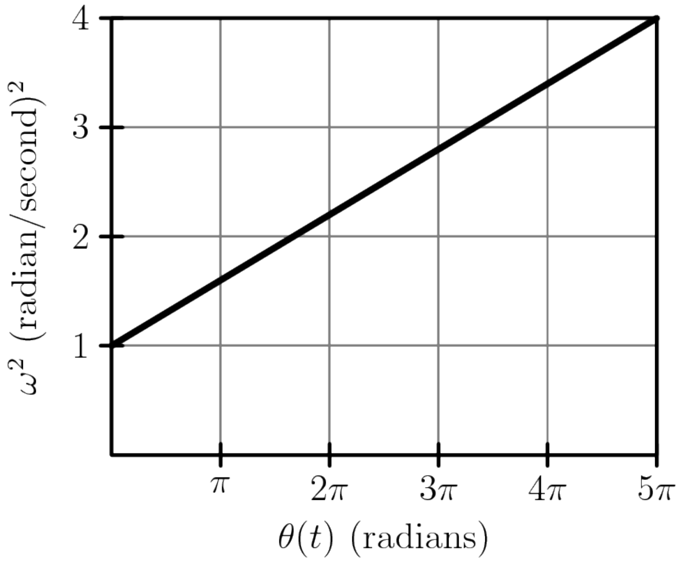
(E)



6. There are two ramps that have an angle of θ_1 and θ_2 respectively. The first ramp has a coefficient of friction that is almost infinite while the second has a coefficient of friction of 0. The ramps are made such that they have the same horizontal length. Two cylinders of equal uniform density and size are placed on both ramps and the time to reach the bottom is measured to be the same. Under these conditions, find the angle θ_2 that minimizes the time for the cylinders to descend the ramps.

(A) 19°
(B) 21°
(C) 30°
(D) 45°
(E) Due to the infinite coefficient of friction in the first ramp, the first cylinder will never descend to the bottom.

7. A gear with moment of inertia 50 kg m^2 is rotating under the influence of an external agent as shown below. A $\omega^2 - \theta(t)$ diagram of the gear's rotation is plotted as shown below. What is the value of the external torque that acts on the gear?



(A) 2.7 N · m
(B) 4.8 N · m
(C) 7.3 N · m
(D) 9.5 N · m
(E) 19.1 N · m

8. A rope is hung between two poles such that it sags down when at rest. The bottom-most point of the rope is grabbed and pulled down such that the sides of the rope become completely straight and make a triangular shape with the poles. How did the center of mass of the rope change?

(A) It moved to a lower position.
(B) It moved to a lower position if the curvature of the rope is initially small, and a higher position if the curvature of the rope was big.
(C) It remains in the same position.
(D) It moved to a higher position.
(E) It moved to a higher position if the curvature of the rope is initially small, and a lower position if the curvature of the rope was big.

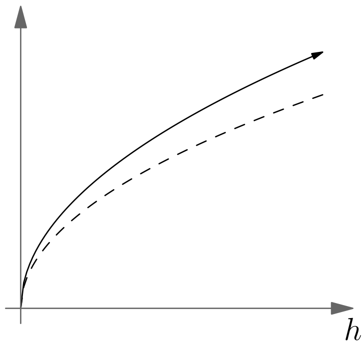
9. A small box A is placed on a floor with coefficient of friction 0.2. A is located a distance of 1 m from an identical box B of the same dimensions. A is then released with a velocity of 3 m/s towards B. Assuming the collision to be entirely inelastic, what is the total distance the box travels from its initial position after it stops?

(A) 1.31 m
(B) 1.45 m
(C) 1.63 m
(D) 1.88 m
(E) 1.96 m

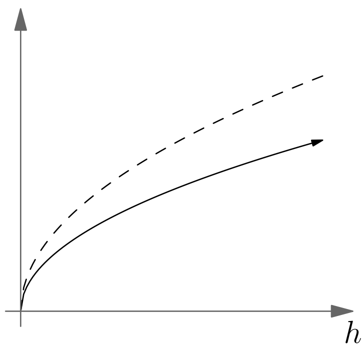
The following information is relevant to problems 10, 11, and 12.

A *snow* cylinder with an initial radius R rolls without slipping down a tall hill with constant slope. As it rolls, snow sticks onto the cylinder which makes its radius slowly increase. The amount of gathered snow is proportional to the distance the snow cylinder has travelled. Consider the following graphs which represent arbitrary units on the y-axis which will be specified according to the problem. (Note: For graph III, the arrow lies directly on top of the dotted line).

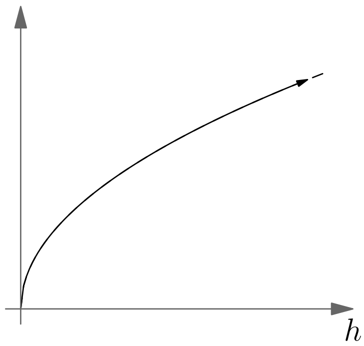
(I)



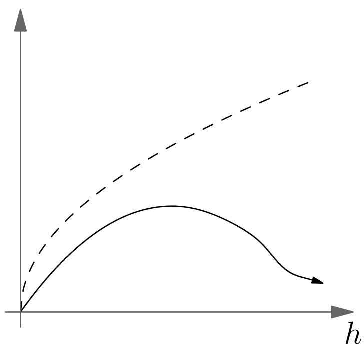
(II)



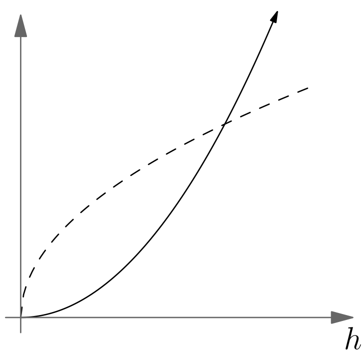
(III)



(IV)



(V)



10. If the dotted line represents the speed of the cylinder if the snow was not sticky, which of the following graphs represents the speed of the cylinder if the snow sticks to the cylinder, as described above?

    (A) Graph I
    (B) Graph II
    (C) Graph III
    (D) Graph IV
    (E) Graph V

11. If the dotted line represents the angular speed of the cylinder if the snow was not sticky, which of the following graphs represents the angular speed of the cylinder if the snow sticks to the cylinder, as described above?

    (A) Graph I
    (B) Graph II
    (C) Graph III
    (D) Graph IV
    (E) Graph V

12. Instead of the cylinder, we roll a large snowball down the hill. Assume the ball travels in a straight line and snow sticks to only the parts of the surface that makes contact with the ground. If the dotted line represents the speed of the ball if the snow was not sticky, which of the following graphs represents the speed of the ball if snow sticks to it?

    (A) Graph I
    (B) Graph II
    (C) Graph III
    (D) Graph IV
    (E) Graph V

13. Books are stacked on each other atop a flat desk that is tilted at an angle of $10°$ with respect to the horizontal. The coefficient of friction between every surface is $\mu = 0.2$ and a book can be thought of as a rectangular prism of length and breadth 10 cm, and thickness of 3 cm. The first book is placed on the desk with an arbitrary orientation with respect to the desk. All subsequent books are placed exactly over the previous book (the resulting stack should be a rectangular prism). What is the maximum number of books we can stably stack on the desk (i.e. the stack does not slide or tip over)?

    (A) 22
    (B) 23
    (C) 24
    (D) 25
    (E) 26

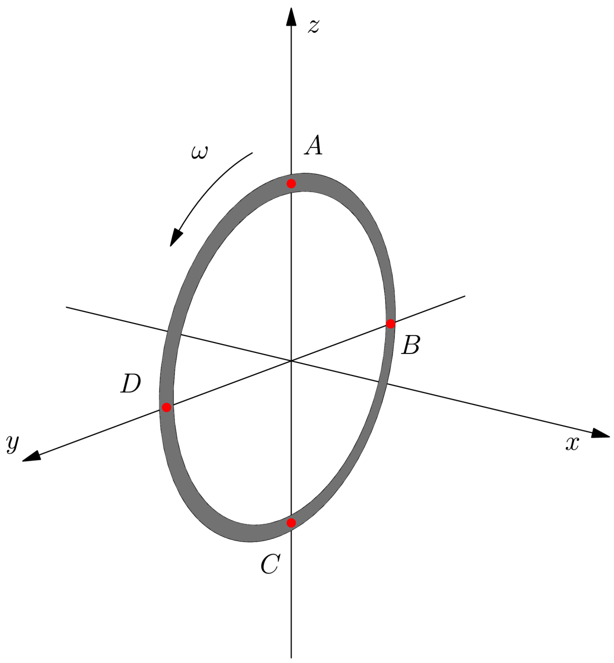
14. A sink is filled with water of density 1000 kg/m^3. A small wooden rectangular block of uniform density 600 kg/m^3 is then placed on top of this water layer. Soon after, a layer of dish soap with density 250 kg/m^3 is poured into the sink until it completely covers the block. Find the ratio of the thickness of the soap layer to the thickness of the block.

    (A) 1/5
    (B) 7/15
    (C) 8/15
    (D) 4/5
    (E) 7/8

15. A planet is able to expand to a radius 2 times its initial radius. Approximately by what factor has the pressure in the atmosphere of the planet changed? Assume that the mass of the planet remains the same before and after the expansion, and no air escapes its atmosphere. Take the universal gravitational constant to be $G = 6.7 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$.

(A) 1/16
(B) 1/4
(C) 2
(D) 4
(E) 8

16. A ring is rotating in the $y - z$ plane as shown in the figure below. The system is isolated and gravity may be neglected. Jack applies a constant force $F\hat{x}$ at some position on the ring which changes the orientation to move along the $x - z$ plane. At which of the following positions was the impulse applied?

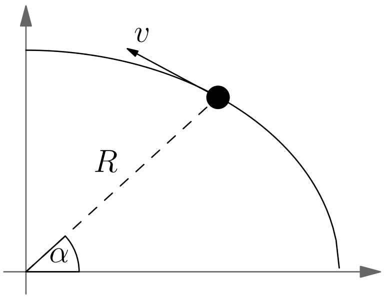


(A) A
(B) B
(C) C
(D) D
(E) The position of the ring will always change irrespective of the position the impulse is applied.

17. A spaceship has a total mass of 500 kg and is moving at a constant speed of 2 m/s. The passengers throw 50 kg of cargo out of the spaceship all at once at a constant speed, causing the spaceship to move at a $37°$ angle with respect to its initial path. Find the minimum speed of the cargo in the frame of the spaceship in order to achieve this course correction. The ship and the cargo can be thought of as point particles.

(A) 8 m/s
(B) 12 m/s
(C) 15 m/s
(D) 18 m/s
(E) 21 m/s

18. A wire is placed in a two dimensional coordinate plane which follows the polar equation $r(\alpha) = \beta e^{\gamma\alpha}$ where r is the distance from the origin to a point on the wire and α is the angle from the origin to the same point. A small bead of mass m is then put onto this wire and the whole system is moved to a vacuum where the effects of gravity are negligible. The bead is then pushed to move on this wire. If the speed of the bead at an arbitrary angle α with respect to the origin and the x-axis is v, find the normal force experienced by the bead from the wire. The distance from the origin and the bead at this moment of time is given to be R.



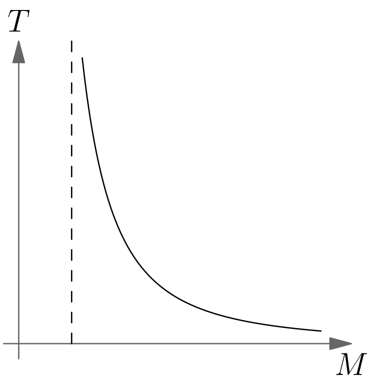
(A) $\dfrac{mv^2}{R\sqrt{1+\gamma^2}}$
(B) $\dfrac{mv^2}{R\sqrt{\beta^2+\gamma^2}}$
(C) $\dfrac{\gamma mv^2}{R\sqrt{1+\gamma^2}}$
(D) $\dfrac{\beta mv^2}{R\sqrt{1+\beta^2}}$
(E) $\dfrac{mv^2}{R\sqrt{1+\beta^2}}$

19. Two identical masses are attached to the same spring and are placed in a vacuum in space. They are made to oscillate at an oscillation rate ω. One of the masses loses its mass at a slow constant rate while the other mass increases its mass at the same constant rate (so the total mass always remains the same). This process happens for some time $\tau < t$ where t is the time for one of the masses to become zero. Which of the following statements is always true about the new oscillation rate Ω?

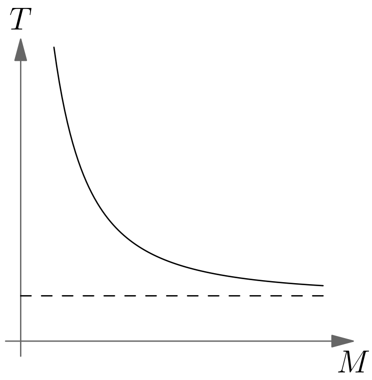
(A) $\Omega < \omega$
(B) $\Omega > \omega$ for short time periods; $\Omega < \omega$ for long time periods.
(C) $\Omega = \omega$
(D) $\Omega < \omega$ for short time periods; $\Omega > \omega$ for long time periods.
(E) $\Omega > \omega$

20. A planet is revolving around a star which has a mass significantly greater than that of the planet. Everytime the planet completes a period, the star has its mass slightly reduced. This process continues repeatedly. Which of the following graphs best represent the mass dependence of the star M to the period of the planet T. *Note:* The x-axis corresponds to a mass that is still much greater than the planet.

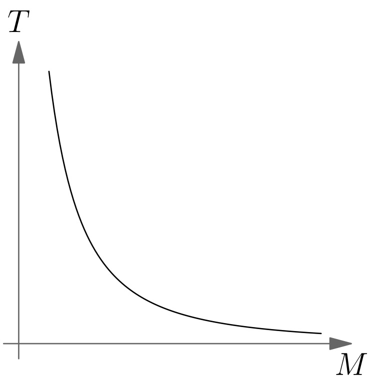
(A)



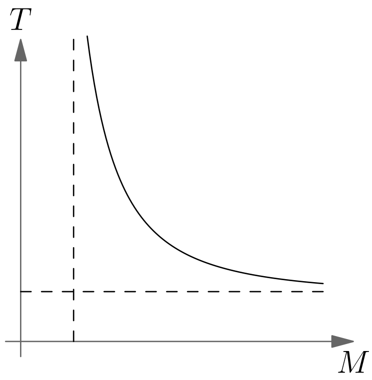
(B)



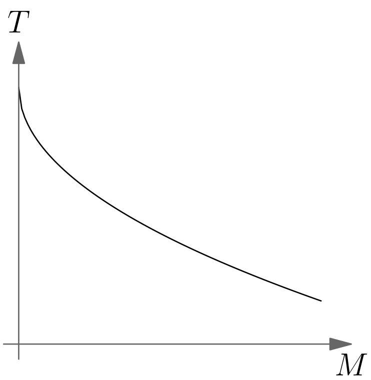
(C)



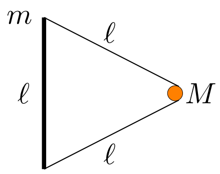
(D)



(E)



21. On a horizontal, frictionless table lies a uniform rod of length $\ell = 1$ m and mass m. A light, elastic thread of relaxed length $\ell_0 = \ell\sqrt{3}$ has its ends attached to the ends of the rod. A small bead of mass $M = m/2$ is embedded on the thread and fixed at its midpoint (the bead is not stuck the cord). The thread is pulled by its midpoint away from the rod until each of its two portions have the same length as the rod, as shown below. Find the distance covered by the bead while it accelerates, after the system is released.

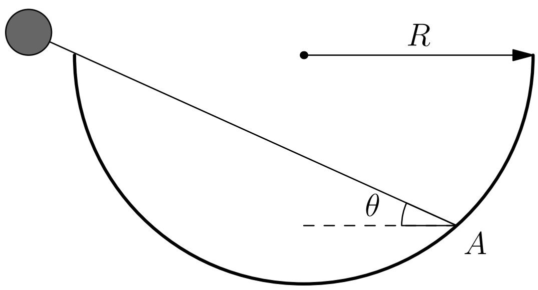


Top view of the system just before it is released.

(A) 0.866 m
(B) 0.106 m
(C) 0.081 m
(D) 0.577 m
(E) 0.244 m

The following information is relevant to problems 22 and 23.

A lollipop is put into a semi-spherical hole of radius R with the inside of its surface having a coefficient of friction of μ. The lollipop consists of a large spherical mass M connected to a uniform rod of mass m and length $2R$. The lollipop makes an angle θ with respect to the horizontal and the hole as shown in the picture and the hole is fixed.



22. What is the minimum value of the absolute coefficient of friction $|\mu|$ of the inner surface of the hole such that the lollipop stays in place?

(A) $\dfrac{M\cos\theta}{M+m}$

(B) $\dfrac{3M}{2(M+m)\sin\theta} + \tan\theta$

(C) $\dfrac{2(M+m)}{5(M+m)\cos\theta} - \tan\theta$

(D) $\dfrac{2M+m}{2(M+m)\sin\theta} - \cot\theta$

(E) $\dfrac{3M+m}{(M+m)\sin\theta} - \cot\theta$

23. At point A, the end of the lollipop is moved by an external source at a constant velocity modulus $|v|$ *along the surface* of the hole. Which of the following expressions best represents the kinetic energy of the mass M as a function of θ? Neglect any frictional effects.

(A) $\frac{1}{2}Mv^2 \sin^2\theta$
(B) $Mv^2 \cdot \frac{1-\cos\theta}{\cos^2\theta}$
(C) $2Mv^2 \sin^2(\theta/2)$
(D) $\frac{1}{2}Mv^2$
(E) $Mv^2(1-\cos\theta)$

24. A solid ball of mass m and radius R lies at rest on a horizontal table in Earth's gravitational field. At $t = 0$, a horizontal force F starts pushing the ball, from a point on its surface situated at height h above the ground, such that the ball is engaged in solely translational motion. After $t = t_1$, the force is no longer applied. Find the number of full rotations will the ball undergo until it starts rolling without slipping.

(A) $\dfrac{3Ft_1^2h^2}{25mR^2(R-h)}$
(B) $\dfrac{5Ft_1^2h^2}{49mR^2(R-h)}$
(C) $\dfrac{3Ft_1^2h^2}{50mR^2\pi(R-h)}$
(D) $\dfrac{5Ft_1^2h^2}{98mR^2\pi(R-h)}$
(E) The ball will never begin to roll without slipping.

25. A city planner decided to conduct a mini study to see how popular a certain road was. He counted the number of cars that passed through the road on two random days. On the first day, he counted 500 cars in 30 min. On the next day, he counted 200 cars in 10 min. The city planner decided to average the rate of car flow on both days. What is the uncertainty he should report?

(A) ± 1 cars/min
(B) ± 3 cars/min
(C) ± 5 cars/min
(D) ± 7 cars/min
(E) ± 10 cars/min