33^{rd} National Semifinal Chinese Physics Olympiad Questions, 2016

PhODS

September 2021

1. (20 points)

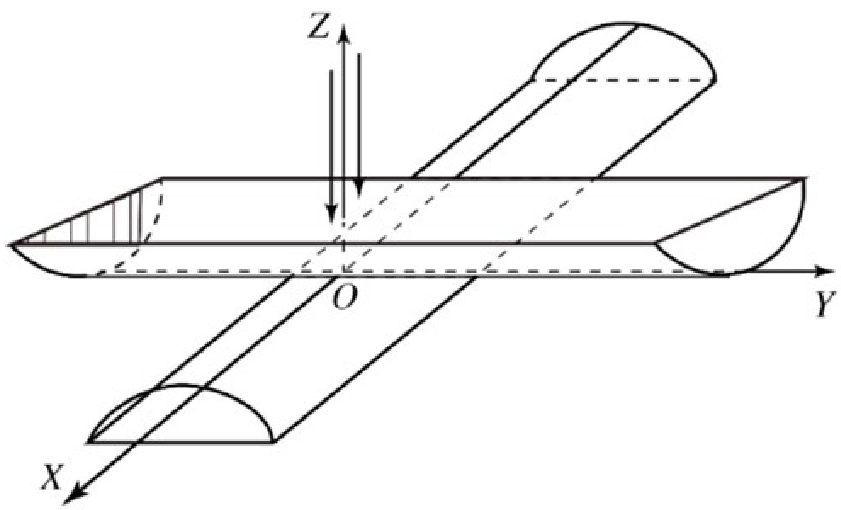


Figure 1: How the cylinder parts look like

In Fig 1-1, two dissected cylindrical parts that is plano-convex and transparent have radii R_1, R_2 respectively, and are tangent to one another. The flat sides of each cylinder parts are parallel to the XY-plane and also parallel to each other. There are 2 imaginary lines that go straight along the top of the curved part of each cylinder, as shown in the diagram (generatrix); both are perpendicular to each other. Take the tangent point O of the two cylinders as the origin of the Cartesian coordinate system O-XYZ. The generatrix of the lower cylinder crossing through O is designated as the x-axis, and the generatrix of the upper cylinder crossing through O is the y-axis. A beam of visible light with wavelength λ in vacuum is travelling in the opposite direction of the z-axis.

When the direction is paraxially incident, the light reflected from the upper and lower cylinders will interfere. With the aid of an optical-reading microscope, opposite the direction of the Z axis, the projection of the interference fringes on the upper cylinder near the origin on the XY plane can be observed. R_1 and R_2 are much larger than the maximum gap between the two cylinders corresponding to the paraxial light interference region. The refractive index of air is $n_0 = 1.00$. Try to derive the equation of the projection of the k-th bright pattern on the XY plane.

Given:

a. On both sides of the interface between the two uniform and isotropic media, the medium with the larger (small) refractive index is an optically dense (sparse) medium; when light is reflected on the surface of an optically dense (sparse) medium , the reflected wave has (non-existent) half-wave loss. In any case, there is no half-wave loss in refracted waves. Accompanied by half-wave loss, a phase mutation of π will be generated.

b. $sin(x) \approx x$, when $x \ll 1$.

2. (20 points)

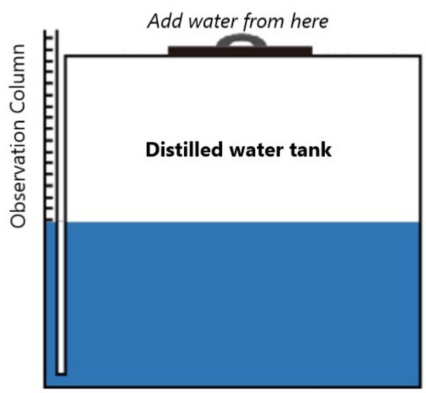


Figure 2: Diagram of the Water Tank

In an early autumn morning, the air temperature was $4.0°C$. A worker went to add water to a cylindrical, stainless steel water tank with an inner diameter of 2.00m and a height of 2.00m. The tank has is a good conductor of heat. There is a transparent cylindrical observation column with an inner diameter of 4.00cm outside the tank, the bottom of which is connected to the tank (the connection is very short), and the top is open to the atmosphere, as shown in the figure. After the water was added, the worker covered a layer of light anti-evaporation film (that is insoluble in water, and produces no friction with the tank wall) on the surface of the water, and sealed the water-filling hole on the top of the tank. At this time, the worker can see that the height of the water in the tank is 1.00m through the scale on the observation column.

(1) From early morning to noon, the air temperature slowly rises to $24.0°C$. What is the water level in the observation column now? Assume that no water occupies the middle connection, the evaporation of water as well as the change in the volume of the tank and observation column with temperature are negligible.

(2) From the time after the water tank is closed to noon, how much work did the air inside the tank do and how much heat did it absorbs? Find the heat capacity of the air in the tank during this process.

It is given that the outside pressure of the tank is always the standard atmospheric pressure of $p_0 = 1.01 \times 10^5$, the density of water at 4.0°C is $\rho_0 = 1.00 \times 10^3 kg/m^3$, the average volume expansion coefficient of water in the temperature change process is $\kappa = 3.03 \times 10^{-4}$, the acceleration of gravity is $g = 9.80 m/s^2$ and absolute zero is $-273.15°C$.

3. (20 points)

Jupiter is the most massive planet in the solar system; its mass is about 318 times that of the Earth. Assume that both the Earth and Jupiter revolve around the Sun in a circular orbit, and the two orbits are in the same plane. Regarding the Sun, the Earth, and Jupiter as point masses, the gravitational forces of other planets in the solar system is ignored, and the gravitational force between the Earth and Jupiter is negligible when there is a Sun. It is known that the masses of the Sun and Jupiter are m_s and m_j respectively, and the gravitational constant is G. The orbital radii of the Earth and Jupiter around the Sun are r_e and r_j respectively. Suppose that at a certain moment, the angle between the Earth-Sun line and the Jupiter-Sun line is θ. If the Sun's mass suddenly becomes zero at this time, find

(1) the velocity of the Earth relative to Jupiter v_{ej} and the minimum velocity v_0 required for the Earth not to be captured by Jupiter's gravity.

(2) Discuss whether the Earth will revolve around Jupiter in the future. You may use your results and data in (1) as well as these: $m_s \approx 2.0 \times 10^{30} kg$, $m_j \approx 1.9 \times 10^{27} kg$, and Jupiter's revolution period $T_j \approx 12\ y$.

4. (20 points)

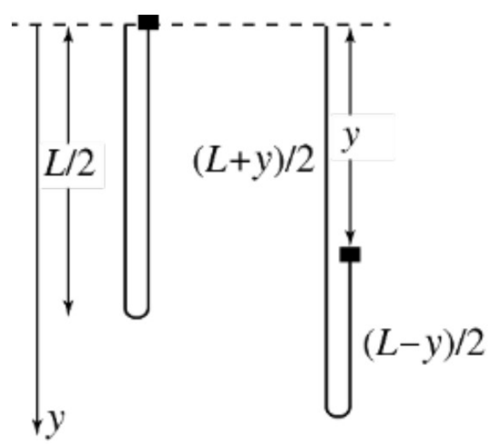


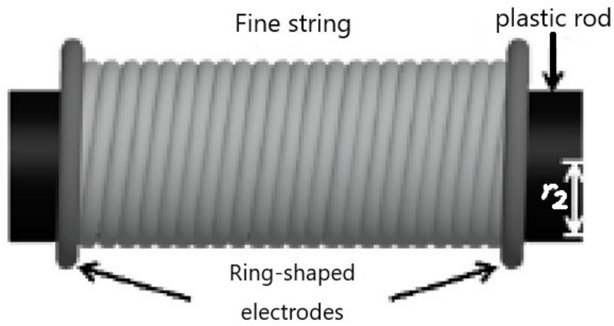
Figure 3: Position of the block when vertical distance fallen is 0 and y

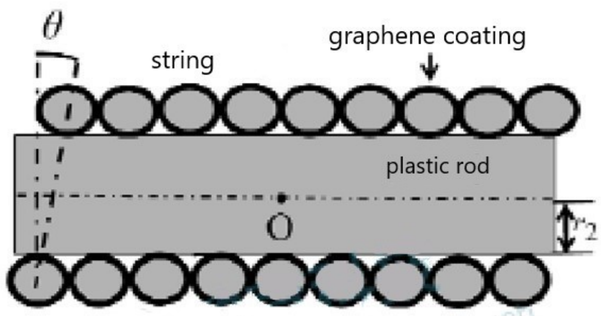
Bungee jumping is a sport that young people love. In order to study the bungee jumping process, a uniform elastic rope with a length of L and a mass of m (which is almost inextensible when only subjected to the weight of the rope itself) is tied to the bridge edge b, and the other end of the rope is tied to a small object with a mass of M. The block is like a bungee jumper. Suppose M is much larger than m, so that the uniform elastic rope will stretch significantly when pulled down by the rope's and the bungee jumper's weight, but still being within the elastic limit. There is no horizontal movement and possible energy loss as the block falls. The magnitude of the acceleration due to gravity is g.

(1) Find the speed and acceleration of the block when the block had fallen a distance y vertically $(y < L)$, and find the maximum speed and acceleration achieved by the block.

(2) Find the tension at the part of the rope that hangs just under b, when the vertical distance is y $(y < L)$.

5. (20 points)





Unstretched rod

How the string might separate after stretching the rod

A schematic diagram of a tensile transducer is shown in Figure a. It is composed of a cylindrical plastic rod with a radius of r_2 and a layer of tightly wounded string that makes N loops $(N \gg 1)$. The string is soft and insulated with a radius of r_1, and the outer surface is evenly coated with graphene of thickness t $(t \ll r_1 \ll r_2)$ and resistivity ρ. Ring-shaped electrodes (that are in close contact with the string) are added to both ends of the transducer.

Before stretching the transducer, the wounded string can be regarded as N oval rings stacked next to one another. The angle between the elliptical ring surface and the "cross section" of the cylindrical plastic rod is θ (see Figure a), and the contact resistance between two adjacent loops of rope is R_c.

Now stretch the entire transducer along the axis of the plastic rod towards both ends. There is exactly one full circle of string in the n gaps between the elliptical rings. This circle of string is adjusted to consist of an opened ring and two short, straight sections (parallel to the axis of the rod) connected in series (see Figure b). Assume that θ, r_1, r_2, ρ, t remain unchanged before and after stretching.

(1) Find the change ϵ in the distance between the ring-electrodes before and after stretching, and the

change in the resistivity of the electrodes;

(2) A current of magnitude I is passed between the two ring electrodes of the sensor. After the sensor is stretched and unstretched, the distance from the midpoint O of the plastic rod on the axis of the plastic rod is D (D is much greater than the length of the sensor) The magnetic induction intensity along the axial direction at point P (not shown in the figure).

When a current of size of I is passed between both electrodes, a magnetic field is produced outside the plastic rod. Find the magnitude of the intensity of magnetic induction at a point D from O that lies on the rod's axis (D is much greater than the length of the rod, so D is a point outside the rod), before and after the plastic rod is stretched.

It is given that the perimeter of an ellipse with a longer axis a and a shorter axis b is $\pi(3\frac{a+b}{2} - \sqrt{ab})$, and that $b \neq 0$.

6. (20 points) A photomultiplier tube is a device used to convert weak light signals into amplified electrical

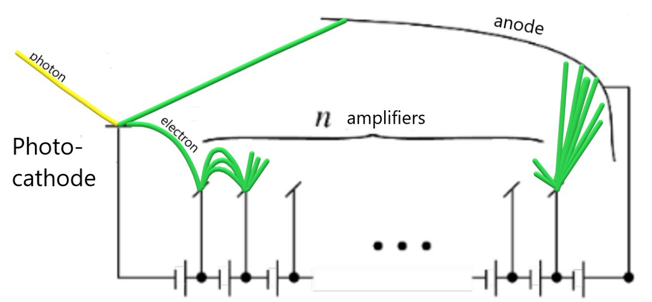


Figure a

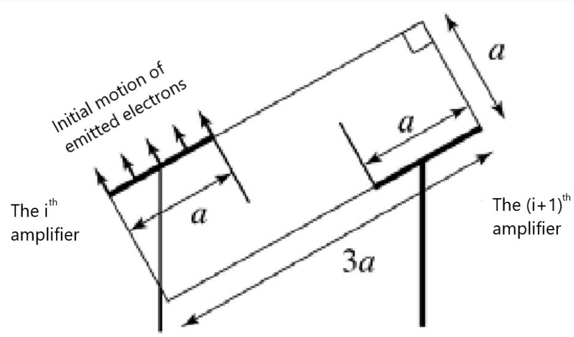


Figure b

signals. A schematic diagram of it is shown in figure a. It is made of a photocathode, n amplifiers and an anode. Each dry cell induces a voltage of V. When a photon hits the photocathode, an electron is emitted, which lands on the 1^{st} amplifier. At each stage, μ % of the electrons that are emitted at the photocathode are absorbed into the first amplifier while the rest are absorbed directly into the anode. Every electron that is absorbed by the $(i+1)^{th}$ amplifier will cause δ more electrons ($\delta > 1$) to be emitted from that amplifier. σ% of the electrons emitted by the i^{th} amplifier are absorbed by the $(i+1)^{th}$ amplifier, while the rest of the electrons are absorbed by the anode. When all the electrons are absorbed by the anode, the signal strength is amplified.

It is given that the electric charge of an electron is e.

(1) Find the average energy of an electron emitted by the photocathode, given that $\delta\sigma > 1$, $n \gg 1$;

(2) In order to make as many electrons as possible from the i-th multiplier stage directly reach the i+1-th multiplier stage instead of the anode, in the early photomultiplier tubes, a uniform intensity perpendicular

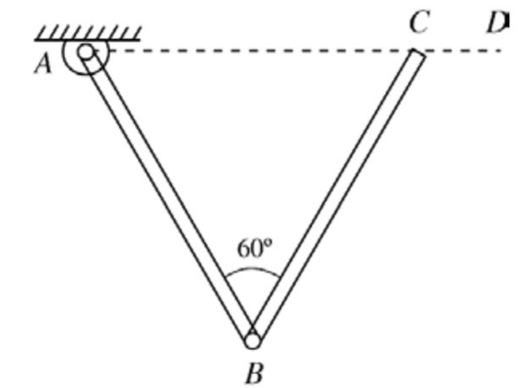
to the plane (paper surface) where the electron trajectory is located is applied. magnetic field. Suppose the length of the multiplication stage is a and the geometric position between adjacent multiplication stages is shown in Figure b, the potential between the multiplication stages

In order for as many electrons emitted by the i^{th} amplifier to reach the $(i+1)^{th}$ amplifier rather than the anode, a uniform magnetic field that is perpendicular to the paper is induced at the location of the photocathode.

Assume that the length of the amplifier is a and adjacent amplifiers are positioned as shown in figure b, and that the electric field produced by the difference in voltage between two adjacent is negligible.

What should be the direction and the strength of the uniform magnetic field, so that as many electrons that is from the i^{th} amplifier and have kinetic energy E_e as possible can directly reach the $(i+1)^{th}$ amplifier? What is the minimum strength of the magnetic field to guarantee that at least some electrons with a kinetic energy of E_e and from the i^{th} amplifier is able to reach the $(i+1)^{th}$ amplifier?

7. (20 points) There are two similar, uniformly thick rods AB and BC, with a mass m, length l. There is



also a fixed hinge at A so that rod AB can swing about it without friction.

Initially, the rods are held stationary by an external force at C. A and C are on the horizontal line AD. A,B, and C lie in a plane vertical to the ground and angle ABC = 60.

The external force on C is then released and the rods start to move within the plane.

(1) If both rods are glued rigid at point B, find

(i) The initial angular acceleration of the system;

(ii) The angular velocity of the rod AB about the point A when the rod AB makes an angle of θ with the horizontal AD.

(2) If B is a frictionless hinge such that BC can rotate about B without resistance, find the initial angular acceleration of both rods as well as the force exerted by the rods on one another.

8. (20 points)

Protons are are made of smaller subatomic particles called partons. The Large Hadron Collider in Europe can make high-energy proton beams collide into one another. The energy of a single proton in a proton beam is $E = 7.0\ TeV\ (1\ TeV = 10^3\ GeV = 10^{12}\ eV)$. Two proton beams with that amount of energy per proton head straight for each other head on, and two partons from two colliding protons interact and annihilate one another to form a new particle.

Suppose that the ratio of the kinetic energy of a and b to the rest-mass energy of the proton is x_a and x_b respectively, and that the kinetic energy of a and b is much larger than the rest-mass energy of a and b.

(1) Suppose that a and b collide, annihilate and form a new particle S with a rest-mass of $m_s = 1.0\ TeV/c^2$. Find the value of $x_a x_b$.

(2) Suppose again that S then decays into two photons. Find the frequency of the two photons in the reference frame of S.

(3) Suppose instead that S decays into two identical lighter particles A with rest-mass $m_A = 1.0\ GeV/c^2$, and then each particle A decays into two photons with identical frequencies (as seen in the reference frame where S is stationary). Find, in this reference frame, the angle between the momentum of the two protons.

It is given that $sin(\alpha) \approx \alpha$, and suppose that $\alpha \ll 1$. The Planck constant is $h = 6.63 \times 10^{-34} J \cdot s$, and the electric charge of an electron is $e = 1.60 \times 10^{-19} C$.