Chinese Physics Olympiad 2017 Finals Theoretical Exam

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Problem 1 (35 points)**.** Refer to Figure 1.1. Balls a and b of masses m_a and m_b respectively are placed on a smooth, electrically insulated, horizontal surface and joined by a light spring of natural length l_0 and spring constant k_0.

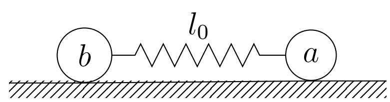

Figure 1.1: Two balls joined by a spring.

(1) When $t = 0$, the spring is at its natural length, a has a rightward velocity of magnitude v_0, and b is at rest. Assuming that the spring experiences elastic deformation throughout the motion, find the time-dependence of the velocities of a and b, $v_a(t)$ and $v_b(t)$ respectively, for all $t > 0$.

(2) Suppose each ball acquires the same charge such that the equilibrium length of the spring is L_0. We denote Coulomb's constant by K. Find the magnitude of said charge q and the frequency f of small oscillations of the system.

Problem 2 (35 marks)**.** Binary star systems are important observation targets for astronomers. We shall model one of these in this problem. Two stars of masses M and m, modelled as point masses, orbit about their barycentre in circular orbits with period T_0. Star M suddenly explodes and loses some of its mass ΔM. We assume that the explosion occurs instantaneously and is isotropic relative to star M, such that the instantaneous velocity of star $M' = M - \Delta M$ after the explosion remains the same as the velocity of M before, and that the explosion and the resulting ejecta have no effect on star m. We are given the gravitational constant G and neglect the effects of general relativity.

(1) Find the distance r_0 between M and m before the explosion.

(2) If M' and m still orbit each other after the explosion, find the period T_1 of this motion.

(3) If M' and m are ejected from orbit as a result of the explosion, derive all the conditions in order for this to be true, in terms of M, m, and ΔM.

Problem 3 (35 points)**.** Children who are well acquainted with the playground swing can force the seat to swing higher and higher by standing up and squatting at appropriate times.[1] A boy of mass m is playing with a swing whose seat and suspension rods (not chains) are light and rigid. We assume that all the boy's mass is concentrated at his centre of mass. When the boy stands, the distance between his centre of mass and the pivot of the swing is l, and when he squats, this distance is $l + d$. Realistically the boy should take a few moments to switch between the two postures, but we will ignore this. We model the situation as follows (see Figure 3.1 for an illustration): The boy squats instantaneously at point A, where he is momentarily at rest, from a standing position. Then he swings to point B, which is lower than A by h_1, at which point he stands up (also instantaneously), such that his final radial speed is zero. Finally, the boy swings to point C, where he is again momentarily at rest. The process repeats in the opposite direction and so on, back and forth, such that the child swings higher with each successive swing. We assume that the force between the child and the seat is parallel to the suspension rods at all times. Take the gravitational potential energy to be zero at B.

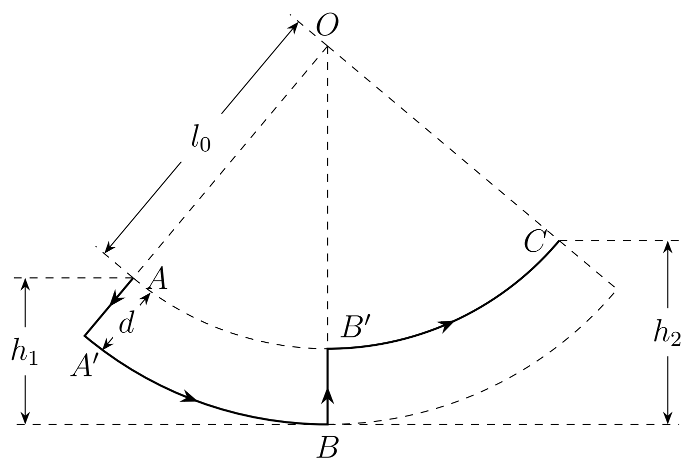

Figure 3.1: A model of a boy on a swing. The solid lines indicate the trajectory of the boy's centre of mass.

(1) Assuming that no mechanical energy is lost as the boy *maintains* his posture, e.g. as he squats from A' to B or stands from B' to C, find the boy's mechanical energy during each of the four stages of his motion, $A \to A' \to B \to B' \to C$. Find also the change in his mechanical energy from A to C.

(2) We now attempt to model the mechanical energy loss even as the boy maintains his posture. We assume that the relationship between the energy loss ΔE and the *absolute value* of the change in height Δh is given by

$$\Delta E = \begin{cases} k_1 mg(h_0 + \Delta h) & \text{when the boy is squatting, and} \\ k_2 mg(h_0' + \Delta h) & \text{when the boy is standing,} \end{cases}$$

where $0 < k_1, k_2 < 1$, h_0, and h_0' are constants and g is the gravitational acceleration. Taking the horizontal at B as the ground, find

[1] *Translator's note.* It was common for Chinese children (and perhaps, even now, daring mad lads both in China and elsewhere) to stand, rather than sit, on the seats of playground swings. Due to sanitary and safety concerns, we are not recommending that you try this at your local playground.

(i) the relationship between h_n, the boy's distance from the ground when he becomes momentarily at rest for the nth time (counting point A as the first time, as shown in Figure 3.1), and h_{n+1}, the distance for the $(n+1)$th time; and

(ii) the relationship between h_n and h_1 and between $h_{n+1} - h_n$ and h_1.

Problem 4 (35 points)**.** Refer to Figure 4.1. A square wire loop *abcd* having uniform density, mass m, side length l, and resistance R lies in a uniform magnetic field of magnitude B which points vertically upwards. The loop can rotate freely about the axis OO', which passes through the midpoints of sides *ad* and *bc*. The two ends of the loop are connected to the leads P and Q. OO' and the x-axis are coplanar and orthogonal. We neglect the self-inductance of the wire loop.

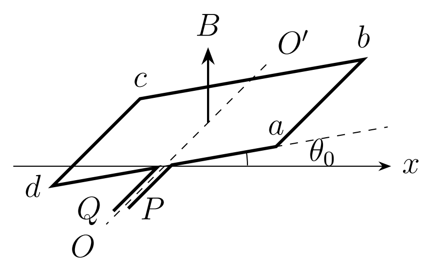

Figure 4.1: A spinning wire loop.

(1) Find the moment of inertia J of the wire about OO'.

(2) When $t = 0$, the wire is at rest and its plane makes a small angle θ_0 with the x-axis. At this instant, we force a steady current I through the wire in the direction given by $P \to a \to b \to c \to d \to Q$. Find the subsequent relationship between θ, $\dot{\theta}$, and $\ddot{\theta}$, where θ is the angle between the the plane of the wire and the x-axis.

(3) When $t = t_0 > 0$, the wire reaches a horizontal position again, whereupon we disconnect the current source between leads P and Q. Describe the subsequent motion of the wire. Hence, derive an expression for the potential difference V_{PQ} between P and Q.

(4) We allow the wire to undergo the above motion for a period of time. We short leads P and Q when the angle θ_1 between the plane of the wire and the x-axis is somewhere within the range $\pi/4 \leq \theta_1 \leq 3\pi/4$. The wire will come to rest after rotating through an angle α. Find the relationship between α and θ_1. Find also the minimum value of α.

Problem 5 (35 points)**.** Refer to Figure 5.1. A thin disc of radius R is placed in the xy-plane such that its centre is at the origin O. The region above the xy-plane (i.e. $z > 0$) is filled with a uniform electric field with magnitude E pointing in the $-z$ direction, whereas the cylindrical region bounded by the xy-plane below and the cylinder with infinite length whose base is said disc (i.e. $z > 0$ and $x^2 + y^2 < R^2$) is filled with a uniform magnetic field B pointing in the $+z$ direction. The region outside this cylinder has zero magnetic field. Now suppose we fire particles carrying charge q, mass m, and speed v from O in all directions above the xy-plane in an isotropic manner (i.e. the probability of being fired in a certain direction is the same regardless of said direction). We ignore the effects of gravity and the interaction between the charges.

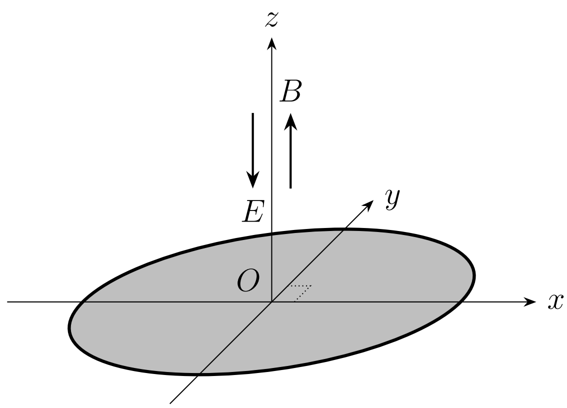

Figure 5.1: A disc suspended along the boundary of an electromagnetic field.

(1) Suppose that all collisions of the charges with the disc are elastic, and that $\eta = 50\%$ of the charges are constrained by the electric and magnetic fields to remain within the cylindrical region. Find the radius R of the disc.

(2) We now introduce a model of non-elastic collisions. Suppose that, when a charge collides with the disc, the direction of the perpendicular component of its velocity is reversed, while that of the parallel component is unperturbed. Suppose further that the magnitudes of both components are reduced by the same proportion such that the kinetic energy of the charge is reduced by 10%.

  (i) Consider the projection of the charge's location onto the xy-plane. Find the length of the path travelled by the projection during the period between the ejection of the charge and its first collision with the disc.

  (ii) Now consider the charge whose projection traverses the greatest distance as described in (i). Find the distance travelled by the particle from its ejection until it comes to rest on the disc.

We are given the integral

$$\int \sqrt{1+u^2}\, du = \frac{1}{2}u\sqrt{1+u^2} + \frac{1}{2}\ln\left(u + \sqrt{1+u^2}\right) + C,$$

where C is a constant of integration.

Problem 6 (35 points)**.** Consider a capillary tube having length 6.00 cm, inner radius 0.500 mm, and outer radius 5.00 mm.

(1) We fix the tube in a vertical position and inject water into it, such that a small amount of water protrudes from the lower end but does not fall. The column of water above it remains at rest due to the effects of surface tension. Given that the distance between the centre of the upper end of the column and the lowest point of the column is 3.50 cm, find the radius of curvature a of the lower interface between the water and the air.

(2) We remove the water in (1) and submerge 1/3 of the tube in water. An external upward force is applied to the tube such that it remains at rest. Find the magnitude of this force.

We are given that the density of glass is twice that of water, the density of water $1.00 \times 10^3\,\mathrm{kg\,m^{-3}}$, the coefficient of surface tension of water $7.27 \times 10^{-2}\,\mathrm{N\,m^{-1}}$, and the gravitational acceleration $9.80\,\mathrm{m\,s^{-2}}$. We may assume that the angle θ between the upper interface and the glass wall is zero.

Problem 7 (35 points)**.** (1) The equivalence principle, first elucidated by Albert Einstein in 1911, states that the laws of physics in a reference frame with gravity present are the same as those in some accelerating reference frame without gravity, provided that the acceleration is equal to a certain value. We study this principle by considering the following two examples.

(i) When a beam of light travels from a place of low gravitational potential to another place of a high gravitational potential, its wavelength increases. This phenomenon is known as *gravitational redshift*. Now consider a spherical body (say, a planet) with uniform density. Suppose that a beam of light with wavelength λ_0 is emitted vertically upwards from a point source A near the surface of the planet. The light beam is detected by a fixed receiver B vertically above A such that $AB = L$. Find the wavelength λ' of the light detected by B.

We are given the mass of the planet M, its radius R (where $R \gg L$), the speed of light c, and the gravitational constant G. We may assume the weak field approximation applies, i.e. we may freely use the results of the Newtonian theory of gravity.

(ii) Refer to Figure 7.1. Suppose that a box whose length is L is suspended in free space. A laser source A and a receiver B are fixed at the lower and upper ends of the box respectively. When time $t = 0$, the box begins to accelerate from rest with magnitude a along the direction of $\overrightarrow{AB}$, where $aL \ll c^2$. Simultaneously, a laser beam of wavelength λ_0 is emitted from A. Using the results of special relativity, find the wavelength λ'' of the light received at B.

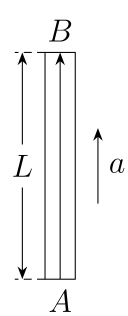

Figure 7.1: A gravity-independent demonstration of gravitational redshift.

(iii) Compare the results of (i) and (ii). How large should a be, in order for us to have $\lambda' = \lambda''$?

(2) Gravitational redshift was first observed by Mössbauer spectroscopy near the Earth's surface, since the Mössbauer spectrometer allows physicists to determine the energy of gamma ray photons to a high precision. The setup is as follows: As in (1)(i), we place a fixed gamma ray source whose frequency is ν_0 at a point A near Earth's surface. We place a Mössbauer spectrometer at point B. We now make the following key assumption: that the spectrometer can only detect photons with frequency ν_0 as measured in the reference frame where it is at rest. In order for the emitted photons to be detectable, the spectrometer must maintain a downward velocity. This experiment was conducted multiple times in a tower of the Jefferson Physical Laboratory at Harvard University by a team led by R. Pound, G. Rebka, and J. Snider from 1960 to 1964, in which $L = 22.6\,\mathrm{m}$. Find the speed of the spectrometer at the instant when the photons from A were detected.

We are given the gravitational acceleration $g = 9.80\,\mathrm{m\,s^{-2}}$ and the speed of light in vacuum $c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$.

Problem 8. A sample of gallium nitride on silicon (GaN/Si) was prepared by depositing a thin layer of gallium nitride uniformly on a silicon wafer, as shown in Figure 8.1.

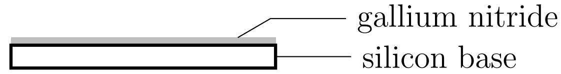

Figure 8.1: A wafer of GaN/Si.

(1) When a beam of light with wavelengths in the range 400 nm–1200 nm is orthogonally incident upon the GaN/Si wafer, and the reflected light measured, we observe that two particular wavelengths are amplified by thin-film interference, one of which is 600 nm. The relationship between the refractive index n of the GaN and the wavelength λ of incident light from a vacuum, i.e. the dispersion relation, is given by

$$n^2 = 2.26^2 + \frac{330.1^2}{\lambda^2 - 265.7^2},$$

whereas the refractive index of silicon is within the range 3.49–5.49. By considering reflection at the GaN surface and the GaN/Si interface, determine the thickness of the GaN layer and the value of the second amplified wavelength.

(2) On the other side of the wafer, two kinds of spectrum-selective materials are coated uniformly on each half of the wafer, as shown in Figure 8.2. For light whose wavelength is a certain value, the coating on the left half is completely absorbent, while that on the right half is completely reflective.

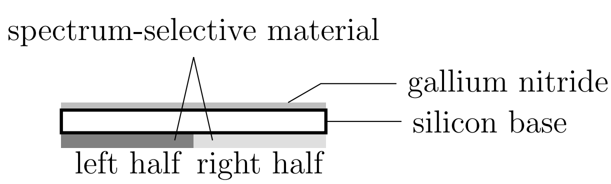

Figure 8.2: A coated GaN/Si wafer.

As shown in Figure 8.3, we suspend the wafer from a fixed support with two strings whose lengths are both a such that the strings are vertical. The wafer has length a and width b, and is allowed to rotate about the axis OO', which causes it to move up and down simultaneously. The wafer is

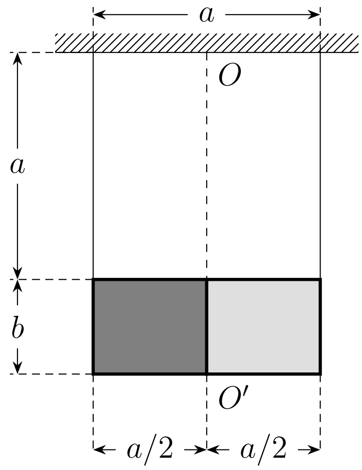

Figure 8.3: A schematic of the suspended wafer.

initially at rest. We shine a strong laser beam whose wavelength has the aforementioned property on the spectrum-sensitive coating such that the light is orthogonally incident upon and uniformly distributed over the surface of the wafer, and wait until the wafer has rotated about OO' to a position where it is stationary again. The direction of the beam remains unchanged throughout the process. The coating is illuminated at all times. We neglect the effects of the radiation on the thin edges of the wafer.

We are given the thickness d' and density ρ' of the silicon wafer, the thickness d and density ρ of the GaN layer, and neglect the mass of the coating. We are also given the vacuum permittivity ε_0 and the gravitational acceleration g. Find the *rms value* E of the electric field associated with the laser such that the wafer rotates through a given angle α.

(3) Taking the values $E = 5.00 \times 10^4\,\mathrm{V\,m^{-1}}$, $d' = 3.00 \times 10^{-4}\,\mathrm{m}$, $\rho' = 2.33 \times 10^3\,\mathrm{kg\,m^{-3}}$, $\rho = 6.10 \times 10^3\,\mathrm{kg\,m^{-3}}$, $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$, and $g = 9.80\,\mathrm{m\,s^{-2}}$, determine the value of α.