CPhO 2018

The following problems were translated by sturdyoak (Jacob Nie).

Problem 1

Suppose the Earth is a sphere with a uniform mass distribution, and the effects of its rotation and atmosphere are negligible. A small object is launched from a space station a height h above the ground with a certain speed relative to the Earth that is perpendicular to the line connecting the center of the Earth and the space station. Given the Earth's radius R, mass M, and gravitational constant G, find the following:

- If the object is to orbit the earth, what conditions must be met by the initial velocity?
- If the initial velocity is v_0 and the ball hits the ground, find the velocity when it hits the ground (magnitude and direction). Find how long it takes to hit the ground.

The following integral is given: for $c < 0$, and $\Delta = b^2 - 4ac > 0$,

$$\int \frac{x\ dx}{\sqrt{a + bx + cx^2}} = \frac{\sqrt{a + bx + cx^2}}{c} - \frac{b}{2(-c)^{3/2}} \arcsin\left(\frac{2cx + b}{\sqrt{\Delta}}\right) + c.$$

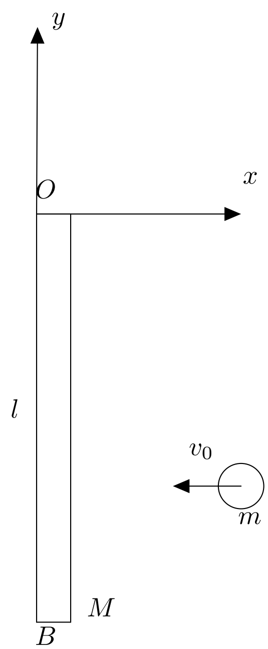
Problem 2

Two thin lenses L_1 and L_2 with focal length f_1 and f_2, respectively, are separated by a distance d and placed coaxially. A ray of light is shone through L_1 and L_2 in that order.

- If the incident ray and outgoing ray are parallel, what condition must the incident ray satisfy?
- Draw all the possible ray diagrams such that the incident and outgoing rays are parallel.

Problem 3

As shown in the figure, a uniform thin rod AB of mass M and length l is freely suspended on a hinge at point O, the origin. (A is not labelled in the diagram, but coincides with O.) The rod is free to rotate in the x-y plane. A projectile of mass m hits the rod at speed v_0 and becomes embedded in the rod. The projectile hits the rod at the strike center, such that the hinge exerts no horizontal force on the rod during the collision. When the rod has swung half a turn to a vertical position, the hinge is magically and suddenly removed. The gravitational acceleration is g pointing downwards, in the negative y direction.



1. Find the distance from the strike center of the rod to O.
2. Find the force of the hinge on the rod when the rod is at an angle $\theta < \pi$ with the vertical, before the removal of the hinge.
3. Let $t = 0$ be the time that the hinge is removed. After the hinge is removed, the position of point B on the rod changes with the time, until it hits the ground. Express the x and y coordinates of B as functions of time. (Recall that the origin is located at point O.)
4. When the rod has rotated another half turn (so now A is once again above B), determine the height difference between points B and O.

Problem 4

The Ioffe-Pritchard magnetic trap can be used to trap atoms, as shown in the figure. The four wires 1, 2, 3, and 4, each with current I, are perpendicular to the x-y plane, and their intersection with the x-y plane is a square with side length $2a$ and center O, where O is the origin. Furthermore, the line joining the intersections of wires 1 and 2 with the x-y plane is parallel to the x-axis, as shown in the figure. The current direction in each wire in indicated in the figure. The entire device is placed in a uniform magnetic field $\boldsymbol{B_0} = B_0\boldsymbol{k}$, where $\boldsymbol{k}$ is the z-axis unit vector. The vacuum permeability is μ_0.

1. What is the magnetic field generated by the current in the wires at the point (x, y)?
2. What is the magnetic field generated by the current near the origin, to first order? (i.e. write your answer to (1) for $x \ll a$.)
3. The binding energy of an atom in the magnetic trap is proportional to the magnitude of the magnetic field at that point. In other words, the potential energy of an atom placed in the magnetic trip is $V = \mu|\boldsymbol{B}_{\text{tot}}|$, where μ is a positive constant. Find the force of the magnetic field on the trapped atom near O to first order.
4. What is the easiest way to remove an atom from the magnetic trap, staying within the x-y plane? Find the minimum kinetic energy for an atom to escape the magnetic trap.

Problem 5

Pieter Zeeman discovered that the sodium spectral D line split into three in a magnetic field. Lorentz explained this according to classical electromagnetic theory, and they won the 1902 Nobel Prize. Assume that the valence electrons in the atom (mass m and charge $-e$, $e > 0$) are subjected to a force $-m\omega_0^2\boldsymbol{r}$, where $\boldsymbol{r}$ is the position vector of the valence electron relative to the center of the atom. The valence electrons go around the atom with angular frequency ω_0 and emit light with angular frequency ω_0. The atom is now placed in a uniform magnetic field directed along the positive z-axis with magnitude $B = \dfrac{2m}{e}\omega_L$.

1. Consider the reference frame that rotates with an angular velocity ω_L in the direction of the magnetic field, such that the valence electrons perform simple harmonic motion in the new reference frame. Find the position of the valence electron as a function of time using Cartesian coordinates, in the new reference frame.
2. Transform the solution of (1) into the original reference frame.
3. Prove that the frequency of the light emitted by the atom in the laboratory reference frame is split into three, and find the interval of frequency between the three frequencies for $\omega_L \ll \omega_0$.

Note: in a rotating reference frame, the electrons are affected by the Coriolis force ($F_C = -2m\omega \times v'$)and the centrifugal force.

Problem 6

There is an experimental device consisting of a inner ball and an outer spherical shell, both concentric, floating in deep space. The space between between the inner ball and the shell is a vacuum. The radius of the inner ball is $r = 0.200$ m, and the temperature is kept constant by some internal means. The emissivity of the ball is $e = 0.800$. The thermal conductivity of the shell is $\kappa = 1.00 \times 10^{-2}\ \mathrm{J \cdot m^{-1} \cdot s^{-1} \cdot K^{-1}}$. The inner and outer radii of the shell are $R_1 = 0.900$m and $R_2 = 1.00$m, respectively, and the outer surface can be considered a black body. The experimental device is at steady-state flow (thermal stability), the emissivity of the shell's inner surface is $E = 0.800$. The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}\ \mathrm{W \cdot m^{-2} \cdot K^{-4}}$, and the cosmic microwave background temperature is $T = 2.73$ K. If the heat transferred between the inner surface of the shell and the outer surface of the shell per unit time is $P = 44.0$ W, then

1. Find the temperature of the surface of the outer shell T_2
2. Find the temperature of the surface of the inner shell T_1
3. Find the temperature of the ball T_0.

You will need to know:

- emissivity: if a blackbody emits radiation at power P, then an object at the same temperature with the same shape will emit radiation at power eP if it has emissivity e.
- Kirchoff's law of thermal radiation: Under steady-state flow conditions, the absorptivity of an object is equal to the emissivity of the object. Absorptivity is the ratio between the power absorbed by an object and the incident power of radiation on the object.
- Fourier's Law: The rate of heat transfer in an object is $-\kappa A \frac{dT}{dz}$, where A is the cross-sectional area.