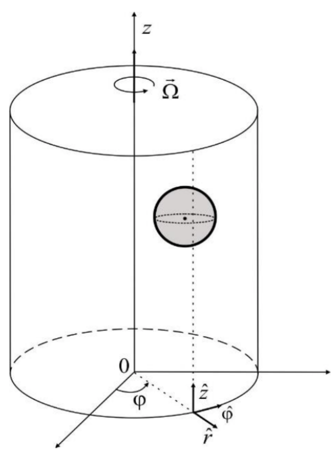
Romanian Extended Team Selection Test for IPhO 2019

Translated By: **Dumbravă Victor-Ioan**
Edited By: **Kushal Thaman**

Problem 1. A ball that's rolling on a vertical wall sometimes seems to have a bizarre behaviour. Golf players are often frustrated by the movement of a ball that just grazes the hole. To simplify the quantitative analysis, the proposed model consists of the following hypotheses:

- the golf ball is considered homogeneous, with mass m and radius r;
- the hole is perfectly cylindrical, of radius R and depth h, measured with respect to the ground;
- the ball rolls without slipping before entering the hole, and it continues to do so on the hole's wall;
- while the ball moves inside the hole, it remains in contact with the wall;
- the ball enters the hole grazingly; its initial velocity is tangent to the hole's wall and its magnitude is v_0.



The figure depicts the ball somewhere on the hole's wall. The azimuthal displacement of the ball is measured by the angle φ, the angular velocity of the ball around the vertical axis of symmetry of the hole is Ω, and the unit vectors of the radial, azimuthal and vertical directions are $\hat{r}, \hat{\varphi}, \hat{z}$, respectively. Under these assumptions, determine:

A. the expression of the vertical component of the angular velocity of the ball, ω_z, when it rolls without slipping on the hole's wall, as well as the expression of the azimuthal component of the friction force between the hole and the wall; **(2.40 p)**

B. the expression of Ω, the angular velocity of the ball around the vertical axis of symmetry of the hole; **(0.40 p)**

C. the expression of the normal reaction force with which the hole's wall acts upon the ball, during its rolling motion on the wall; **(0.40 p)**

D. the vertical position $z(t)$ of the ball as a function of time, after it enters the hole; **(4.60 p)**

E. the expression which gives the minimum value of v_0 such that the ball immediately exits the hole; **(1.00 p)**

F. the mathematical condition which μ (the coefficient of friction between the ball and the hole) has to satisfy, such that the ball never slips. **(1.20 p)**

The value of the gravitational acceleration, g, is known.

Notes:

1. *The derivative of the product of two functions,* $f(x)$ *and* $g(x)$, *is* $\frac{d}{dx}(fg) = \frac{df}{dx}g + f\frac{dg}{dx}$;
2. *The time-derivatives of the unit vectors depicted in the figure are* $\frac{d\hat{r}}{dt} = \Omega\hat{\varphi}$, $\frac{d\hat{\varphi}}{dt} = -\Omega\hat{r}$ *and* $\frac{d\hat{z}}{dt} = \vec{0}$.

Proposed by **conf. univ. dr. Sebastian POPESCU**, Faculty of Physics, ”Alexandru Ioan Cuza” University, Iași

Problem 2. An ideal heat engine has two sources of finite thermal capacity $C = 500\ J/K$, with initial temperatures $T_H = 625\ K$ and $T_C = 289\ K$. The power of the engine decreases exponentially in time according to the law $P = P_0 e^{-k\tau}$, where $P_0 = 960\ W$ is the initial power, and τ is the time elapsed.

A. Calculate the temperatures of the two sources, when the engine ceases to function. **(2.5 p)**

B. Calculate the total work done by the engine. **(1.5 p)**

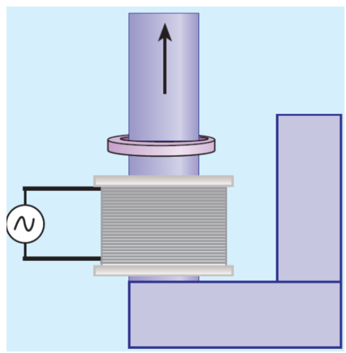
C. Calculate the time required for the power of engine to halve. **(2.5 p)**

D. Calculate the time required for the efficiency of engine to halve. **(2 p)**

E. Calculate the time required for the temperature difference between the sources to halve. **(1.5 p)**

Proposed by **asist. cerc. dr. Gabriel PASCU** and **lect. univ. dr. Adrian NECULAE**, Faculty of Physics, West University of Timișoara

Problem 3. An experiment analyzed thoroughly during the past two decades is known in the scientific community by the name of "jumping ring", first presented in 1887. The analysis of this experiment lead to the emergence of dozens of demonstrative clips, as well as a few scientific articles. The basic scheme of the experiment requires the presence of a coil with an iron core (which is longer than the coil itself), coupled to a voltage source and positioned such that its symmetry axis is vertical. The coil has circular loops, and the iron core has a circular cross section of exterior diameter D. A conducting, non-magnetic ring of interior diameter D, exterior diameter $D+2x$, thickness H and density ρ is placed above the coil, such that it is concentric with the coil and its iron core. The ring can move vertically without friction. Under certain circumstances, the ring suddenly jumps, more or less.

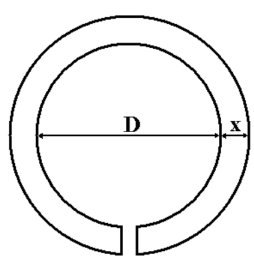


Task 1 (3.5 p):

A. Find the mass of the ring **(0.5 p)**

B. Considering that the coil is connected to a DC voltage source and a switch, analyze the movement of the ring right after the switch is closed, explaining the cause of the phenomenon. **(2.0 p)**

C. What happens if the ring has a small gap, as depicted in the second figure? **(1.0 p)**



Task 2 (3.5 p):

Considering that the coil is now connected to an AC voltage source of angular frequency ω analyze the difference in the behaviour of the ring after the switch is closed, in the following two situations:

D. The iron core causes the magnetic field produced by the coil to be perfectly vertical above the coil (until right below the maximum height of the iron core); **(0.5 p)**

E. The condition stated at **(D)** isn't satisfied (that is, the magnetic field has a horizontal component as well). **(3.0 p)**

Note: *When solving **Task 2**, you need to take the following cases into account – the ring is (1) purely resistive (2) has an inductance L. In order to motivate what you think it's going to happen with the ring, calculate the forces that act upon it, as well as the average forces over a period (of the AC source).*

Task 3 (3.0 p):

F. What would differ regarding the behaviour of the ring, if it were held in liquid nitrogen before performing the experiment? **(1.0 p)**

G. Under the circumstances described at **(E)**, it can be observed that if the iron core is sufficiently long, there exists an equilibrium position of the ring at a certain distance above the coil (which is coupled to the AC source). Find the form of the relation between the inductance of the ring and its thickness, if the equilibrium position doesn't depend on the value of H (assuming that the material of the ring is a very good electrical conductor). **(2.0 p)**

Note: Throughout the problem, air resistance is negligible when analyzing the motion of the ring.

Proposed by **lect. univ. dr. Mihai VASILESCU**, Faculty of Physics, "Babeș-Bolyai" University, Cluj-Napoca

Problem 4. *Magnetic field appears near moving charges. At a point in space, a magnetic field exerts a force on a moving charge located at that point.*

An infinitesimally small segment of a conductor (a current I flows through it), of length $d\ell$, produces – at a point P located at a distance $\vec{r}$ from it – a magnetic field of strength (given by the Biot-Savart law) $d\vec{B} = \frac{\mu_0 I d\vec{\ell} \times \vec{r}}{4\pi r^3}$; in the plane that passes through point P and is perpendicular to the direction of the conductor $d\vec{\ell}$, the magnetic field lines are circular.

The magnetic field has no sources; the field lines are closed. For permanent magnets – objects with magnetic properties – there exists a point through which field lines (which are closed lines) exit the object, called north pole, and a point through which field lines enter the object, called south pole. Permanent magnets are treated - due to their pole description – as dipole structures. By analogy with the electric dipole, a magnetic dipole moment can be defined.

The magnetic field has no sources; the field lines are closed. For permanent magnets – objects with magnetic properties – there exists a point through which field lines (which are closed lines) exit the object, called north pole, and a point through which field lines enter the object, called south pole. Permanent magnets are treated - due to their pole description – as dipole structures. By analogy with the electric dipole, a magnetic dipole moment can be defined.

The magnetic field generated by a permanent, cylindrical magnet is equivalent – at very large distances – with the magnetic field produced by a circular loop through which a steady current flows. The loop is characterized by its magnetic dipole moment $\vec{p}_m$. The magnitude of this vector is defined as the product between the intensity I of the current that flows through the loop and its surface area S: $p_m = I \cdot S$.

A force $d\vec{F} = I \cdot d\vec{\ell} \times \vec{B}$ acts upon an infinitesimal segment $d\vec{\ell}$ of a conducting wire, through which a current I flows, and which is located at a point where a magnetic field of strength $\vec{B}$ exists. The change in position of a circular loop in a uniform magnetic field $\vec{B}$ requires some mechanical work, which represents the change in its potential energy. The potential energy U associated with an object of dipole moment $\vec{p}_m$ has the expression $U = -\vec{p}_m \cdot \vec{B}$.

If you deem it necessary, you can consider that the equation of motion for damped oscillations, $\ddot{x} + 2 \cdot \beta \cdot \dot{x} + \omega_0^2 \cdot x = 0$, has the solution $x(t) = A \cdot \exp(-t/\tau) \cdot \cos(\omega \cdot t + \varphi)$, where $\omega = \sqrt{\omega_0^2 - \beta^2}$ is the angular frequency of the damped oscillations and $\tau = 1/\beta$ is the damping time. The parameters A, φ can be determined from initial conditions.

Task 1:

For **Task 1**, consider a magnetic dipole consisting of a circular current loop. The magnetic moment of this dipole has strength p_m. The axis of the dipole is perpendicular to the plane of the loop and it passes right through its center. Because magnetic fields have no sources, the number of field lines that cross any closed three-dimensional surface is null.

1.A. Prove that the strength of the magnetic field created by the dipole at a point A, located on the axis of the dipole at a large distance from its center has the expression

$$B_z = a\frac{p_m}{z^\alpha} \tag{1}$$

where z represents the coordinate of point A, measured along the axis of the dipole, with respect to its center. Deduce the values of a and α, which appear in the expression from relation (1). Assume that the magnetic dipole is placed in vacuum ($\mu_0 = 4\pi \cdot 10^{-7}\ Hm^{-1}$). **(2.0 p)**

1.B. Next, consider that the aforementioned magnetic dipole is placed in a magnetic field with axial symmetry, but which is inhomogeneous, whose strength along the Oz axis is $B_z(z)$. If the axis of the dipole coincides with the symmetry axis of the magnetic field, prove that the expression of the force that the magnetic field exerts on the dipole is

$$F_z = c \cdot p_m \cdot (dB_z(z)/dz) \tag{2}$$

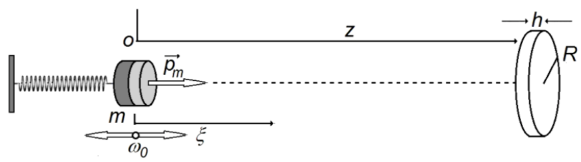
and determine the value of the constant c. **(1.5 p)**

Task 2:

In **Task 2**, you will analyze the oscillations of a permanent, cylindrical magnet of mass m and magnetic dipole moment p_m. The magnet is fixed at one end of a spring with constant k. The magnet can oscillate along a horizontal axis, that has the same orientation as the magnetic moment. You will use the letter ξ to denote the elongation of the oscillation. The other end of the spring is fixed. All friction forces are negligible.

2.A. Determine the expression of the angular frequency ω_0 that corresponds to the small oscillations of the magnet-spring system, in the absence of any external magnetic fields. **(0.5 p)**

Now, consider that a small, metallic disk is placed at a distance z from the equilibrium position of the magnet, in such a way that the symmetry axes of the magnet and the disk coincide (as illustrated in the figure, which isn't drawn to scale).



The disk has radius R and thickness h ($h \ll R \ll z$) and is held fixed. The material of the disk has electrical conductivity σ and relative permeability $\mu_r \simeq 1$. The magnet is given a small initial displacement from the equilibrium position and is then released, causing it to undergo small, lightly-damped oscillations, described by the law $\xi(t)$, where $\xi \ll z$.

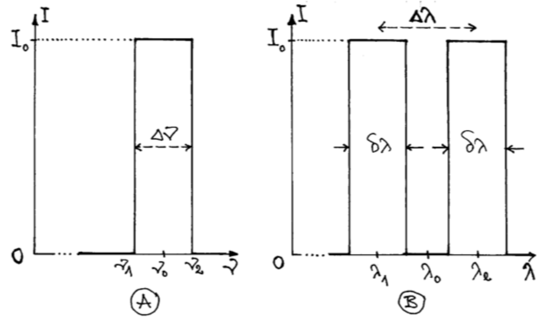
2.B. Deduce the expression of the force F which the metallic disk exerts onto the magnet, that undergoes small, lightly-damped oscillations. Express the result in terms of the speed of the magnet, $v = \dfrac{d\xi}{dt}$, and other quantities specified in the problem body. **(4.0 p)**

2.C. Write the equation of the magnet's oscillation, under the conditions specified at **(2.B.)**. **(1.0 p)**

2.D. Deduce the expression for the relative change of the angular frequency, $\dfrac{\Delta\omega}{\omega_0}$, in the presence of the metallic disk, with respect to the angular frequency of the free oscillation. **(1.0 p)**

© Proposed by **conf. univ. dr. Cristina MIRON** and **conf. univ. dr. Adrian DAFINEI**

Problem 5. A. An interferometer with two waves (of the same amplitude) is illuminated with the red radiation of Cadmium, having wavelength $\lambda_0 = 6438.8$Å and spectral linewidth $\Delta\lambda = 1.2 \cdot 10^{-3}$Å. On the frequency scale, the distribution of the spectral intensity of the radiation has a rectangular profile, as depicted in the figure.



The difference in optical path between the two waves that interfere at the observation point M is denoted by L, and $p \equiv L/\lambda_0$ is the order of interference at that point.

1. Express the luminous intensity at point M as $I(M) = 2I_0[1 + V(p)\cos(2\pi p)]$ and explain - in terms of $\lambda_0, \Delta\lambda$ and p - the significance of the **degree of coherence (visibility)** $V(p)$. **(1.50 p)**

2.A. Determine the **coherence length** $\Delta\ell$, defined as the lowest value of the optical path difference for which the fringes of interference completely disappear (can't be observed anymore). Show that the **time of coherence**, $\Delta t = \Delta\ell/c$, is equal to the inverse of the frequency band $\Delta\nu = \nu_2 - \nu_1$, that is, $\Delta t = 1/\Delta\nu$. **(1.50 p)**

2.B. Determine the maximum value of the order of interference p, such that the visibility $V(p)$ of the observed fringes exceeds 90 %. Take into account that $\sin 45° \approx 0.225\pi$. **(0.75 p)**

B. The experiment is carried out once again. Now, the interferometer is illuminated by a lamp with sodium vapours, which emits two types of radiation of equal intensities (I_0), of wavelengths $\lambda_1 = 5980$Å and $\lambda_2 = 5896$Å and equal spectral linewidth $\delta\lambda = 0.11$Å. Let the arithmetic mean of λ_1 and λ_2 be λ_0.

1. Express the luminous intensity at point M in a similar manner to **(A.1)** and determine the corresponding degree of coherence $V(p)$, in terms of the order of interference $p = L/\lambda_0$, λ_0, $\delta\lambda$ and $\Delta\lambda = \lambda_2 - \lambda_1$. Trace the graph of $V(p)$ (qualitatively). **(3.25 p)**

2.A. What is the order of interference p_0 that maximizes the luminous intensity at point M? **(0.50 p)**

2.B. What is the order of interference p_1 such that the luminous intensity at point M halves for the first time, compared to the value it reaches under the circumstances of **(B.2.A)**? **(0.75 p)**

2.C. For which order of interference p_2 does the contrast of the fringes start to increase again, after having decreased continuously? **(0.75 p)**

Proposed by **prof. univ. dr. Florea ULIU**, University of Craiova.