RuPho final round, 11 grade. Translations and solutions

Nicov Stefan

Octomber 2021

1 Problems 1992

11.1 A small ball is suspended from a beam on a thin weightless thread of length $l = 0,1m$. At what minimal speed v_0 we should launch the ball in the horizontal direction so that it hits the beam at the suspension point?

11.2 Suppose that a future spaceship can withstand any thermal and mechanical overload. Find the minimal period of revolution around Sun (and arguments why this period is minimal) knowing that the angular size of the Sun, seen from Earth, is $\alpha = 9,3 \cdot 10^{-3}$ radians.

11.3 In a thermally isolated cylinder, the piston of which is held stationary by two identical masses, there is 1 mole of monatomic ideal gas. The initial temperature of the gas is T_0. There is no pressure outside the cylinder. How will the temperature of the gas change if we remove one of the masses, and after a certain time, we put it back?

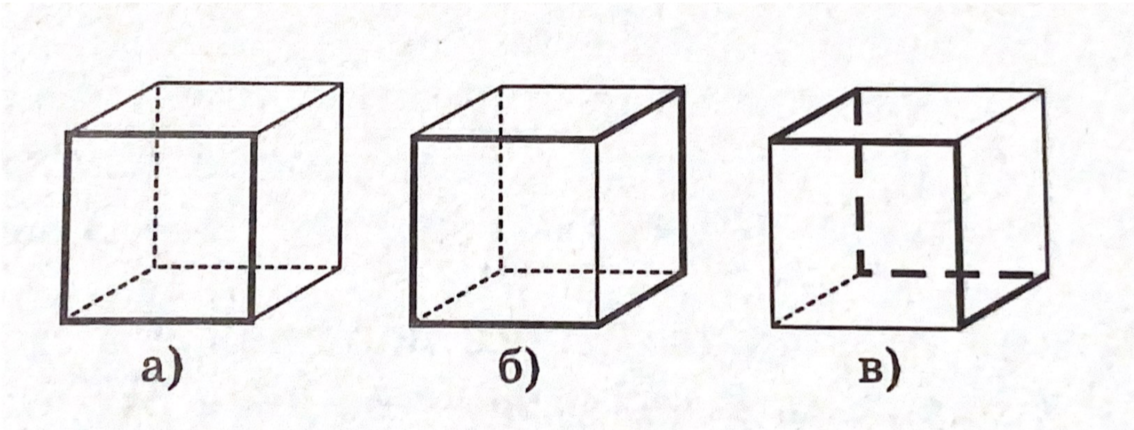
11.4 Consider the setup of three small identical metal balls arranged in a regular triangle. The entire system is in a vacuum. The balls are alternately connected one time at a time to a remote conductor, the potential of which is kept constant. As a result, the first ball has a charge Q_1, the second ball has a charge Q_2. Find the charge of the third ball.

11.5 The mass of Charon, the recently discovered satellite of Pluto, is 8 times less than the mass of the planet. Pluto and Charon revolve in circular orbits around a common center of mass, and they look at each other all the time, i.e. the system rotates as a single rigid body. The distance between the centers of the planet and the satellite is $R = 19640$ km, Charon's radius is $r = 593$ km. Determine the relative difference in the acceleration of gravity on Charon at the point closest to Pluto and at the point farthest from it.

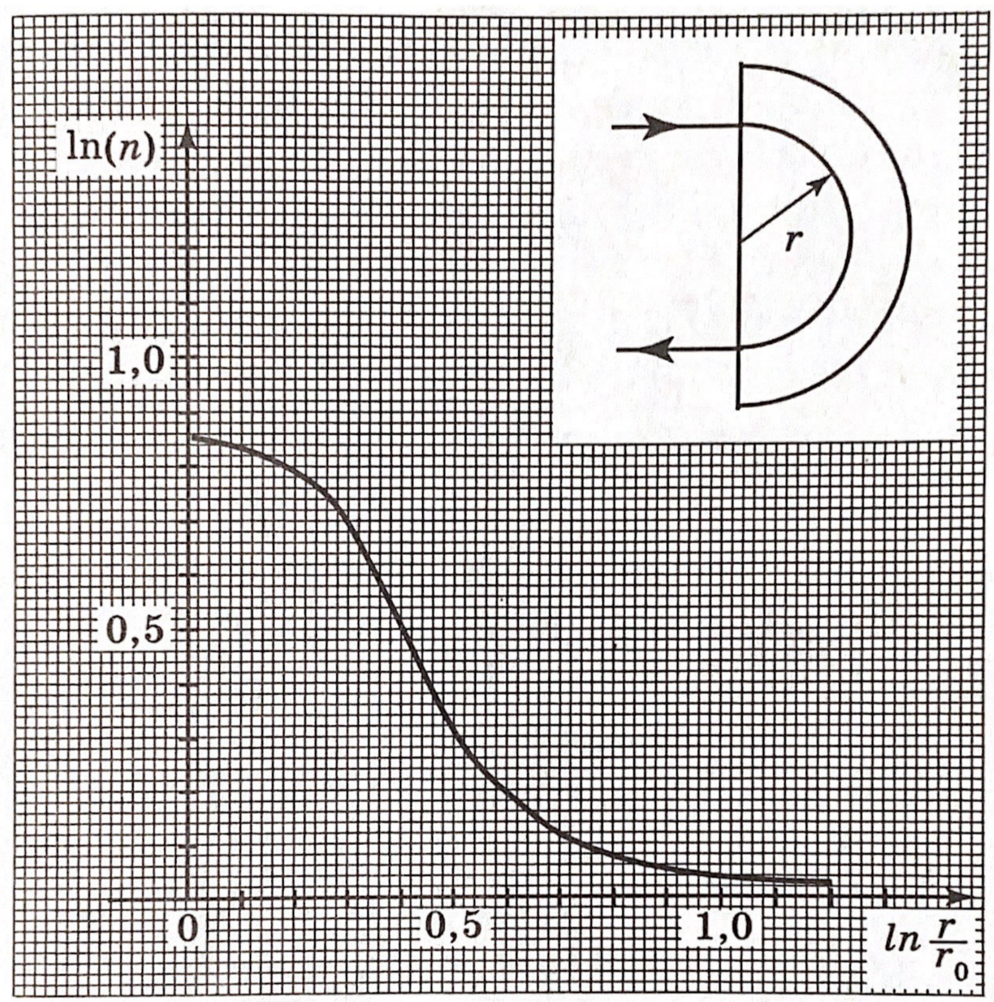
11.6 One of the thermocouple junctions is at room temperature $T_1 = 300K$, and the other is in a thermally isolated vessel with ice at temperature $T_2 = 273K$. The power developed by the thermocouple is released on the resistance of the heater, which is placed in another thermally insulated vessel containing water. Estimate the rise in water temperature by the time the ice melts. The mass of ice and water is the same. Specific heat of water $c = 4200$ J/(kg K) and latent heat of letting ice is $\lambda = 335000$ J/kg.]

11.7 A charged particle, moving in a plane perpendicular to a long uniformly charged wire, flies past this wire, deviating from the original direction by a small angle α. Find this angle if the kinetic energy of the particle when it enters the field of the wire is W, it's charge is e, linear charge density of the wire is q. The field at a distance R from the long wire is $E = \frac{q}{2\pi\varepsilon_0 R}$.

11.8 A coil of a thin wire, in the shape of a square, has an inductance L_1. A loop of the same wire running along the edges of the cube, as shown in the second figure, has an inductance L_2. Find the inductance of the coil in the third figure if it's made from the same wire. The coils in the figures are highlighted.



11.9 The half-cylinder is made of an optically transparent material with a radially variable refractive index n. The dependence of n on the radius r is shown on the graph in coordinates $\ln n$ and $\ln \frac{r}{r_0}$ where $r_0 = 1$ cm. Using this dependence, find the radii of the semicircles, along which a thin beam of light can propagate when it normally falls on the flat surface of the half-cylinder.



2 Solutions (Partial)

11.1 The tension of the thread is $T = mg\cos\alpha + \frac{mv^2}{l}$ and it becomes zero when

$$\cos\alpha = -\frac{v^2}{gl}$$

Writing the energy conservation, we find that $\frac{mv_0^2}{2} = \frac{mv^2}{2} + mgl(1-\cos\alpha)$ or equivalently

$$\cos\alpha = -\frac{v_0^2 - 2gl}{3gl}$$

Next, the ball moves to the suspension point on a parabola:

$$x = -v\cos\alpha t$$

$$y + v\sin\alpha t - \frac{gt^2}{2} = 0$$

where $x = l\sin\alpha$ and $y = -l\cos\alpha$. Then we find two equations for the flight time t.

$$t = \frac{v\sin\alpha + \sqrt{v^2\sin^2\alpha - 2gl\cos\alpha}}{g}$$

$$t = -\frac{l}{v}\tan\alpha$$

Equating them, and using the fact that $-gl\cos\alpha = v^2$ we get

$$v\sin\alpha + \sqrt{v^2\sin^2\alpha + 2v^2} = v\frac{\sin\alpha}{1-\sin^2\alpha}$$

.

The solution is $\sin^2\alpha = \frac{2}{3}$, or $v_0 = \sqrt{gl(2+\sqrt{3})} \approx 1.93$ m/s.

11.2 From Kepler's third law, we know that $\frac{T_1^2}{a_1^3} = \frac{T_2^2}{a_2^3}$. Then, of course, the minimum period of revolution will be in a circular orbit around the Sun with the radius equal to that of the Sun itself. $R_S = \frac{\alpha}{2}R_E$ where R_E is Earth-Sun distance, R_S is Sun's radius. Putting all this together, we get $T_{min} = T_E(\frac{\alpha}{2})^{\frac{3}{2}}$ where $T_E = 1$ year is Earth's period of revolution.