81st Moscow Olympiad for Students in Physics 2020
Grade 11, Round 1

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Problem 1. Capacitors (7 points)

In the circuit given in the figure, initially the capacitor with a capacity $3C = 300\ \mu F$ is charged up to $U_0 = 12\ V$, meanwhile the other two capacitors with capacities C and $2C$ are uncharged. The switch P is in the "middle" position.

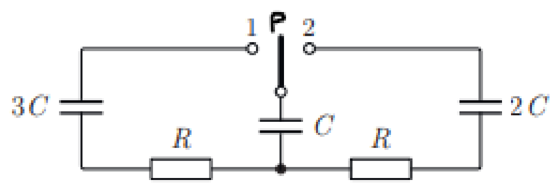

Figure 1: Problem 1

First, the switch is put into position 1 for a small amount of time (much smaller than RC), then, into position 2 for a much longer period of time. Find the charges on capacitors after these two procedures are repeated a large number of times. Find the approximate amount of heat dissipated in each of the resistors.

Problem 2. A Vessel in Ice (8 points)

An airtight metallic vessel is filled with a mix of air and water vapour. When placed in a thermostat with melting ice, it starts cooling down. During the process, the measurements are taken with error in temperature $\Delta T = 0.5°\text{C}$ and in pressure $\Delta p = 0.05 \cdot 10^5$ Pa. Results of these measurements are given in the following table:

t, °C	137	123	109	82	55	27	0
p, 10^5 Pa	1,5	1,45	1,4	1,3	0,8	0,7	0,6

Figure 2: Problem 2

Find the ratio of the amount of water to the amount of air in the vessel. Also find the density of the gaseous phase at the start and the end of the process. Take into account the fact that the pressure of 1 kPa of saturated water vapour is reached at a temperature of about 7°C. Molar masses of water and air are 18 g/mol and 29 g/mol, respectively.

Problem 3. Image in a ball (9 points)

An observer sees the reflection of the Sun in a polished metallic ball. Angular height of the sun relative to the horizon is α and is equal to the angle between the line of sight and the horizontal normal to the ball. Determine the size of the image of the Sun if the radius of the sphere is R and the angular size of the Sun is ϕ ($\phi \ll \alpha$).

Note - For a small angle ϕ the following approximations are true: $\cos\phi \approx 1, \sin\phi \approx \phi$.

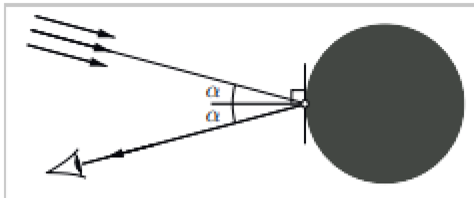

Figure 3: Problem 3

Problem 4. Models of a Starter (12 points)

The figure shows the simplest connecting an electric motor (car starter) to a battery.

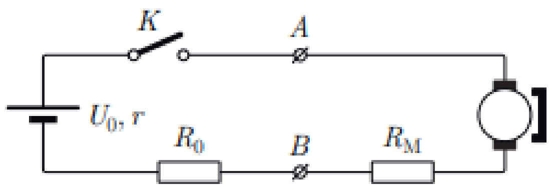

Figure 4: Problem 4

The circuit parameter $R_M = 2 \cdot 10^{-2}$ Om regulates the resistance of the motor armature windings, r and R_0 are the internal resistance of the battery with EMF $U_0 = 12$ V and the wires, with $R_0 + r = 10^{-2}$ Om. It can be assumed that the induced EMF generated by the electric motor is proportional to the angular speed of rotation of the shaft $|\varepsilon_i| = k\omega$, and the moment of forces acting on the shaft due to the magnetic field is proportional to the current $M = kI$. To simplify the calculations, we assume that the motor shaft is not loaded.

1. Find the voltage U_{AB} across the motor terminals (between points A and B) immediately after the key is closed, as well as the maximum value of the current in the circuit. What is the current at the moment when the angular speed of rotation of the shaft is 75% of its maximum value? *(3 points)*

   Another model (Fig. shown below) takes into account the presence of inductance in the structure of the induction motor, R_l is the resistance of the coil of inductance L. When being switched on, the key K_1 is closed first, and when the current through the coil has reached its steady-state value, the key K_2 is closed.

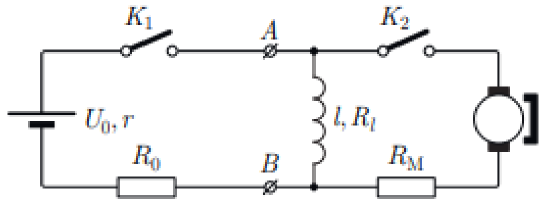

Figure 5: Problem 4

2. Assuming that the ratio $\alpha = \dfrac{R_l}{r + R_0}$ is known ($\alpha > 1$), determine the maximum current through the motor in this case. *(4 points)*

3. When turning off the electric motor (after the angular speed of the shaft has reached a steady state value), first the key K_1 is opened. In this case, the voltage across the motor terminals almost instantly increases by $\Delta U_{AB} = 2$ V. Determine the parameter α from this data. *(3 points)*

   The circuit shown in the third figure here is closest to the real device. The keys are initially open. Switching on the ignition key corresponds to closing the key K_1. When the current through the coil L reaches a certain threshold value I_p, the key K_2 is closed (the magnetic field of the coil pulls in the rod that closes the key K_2).

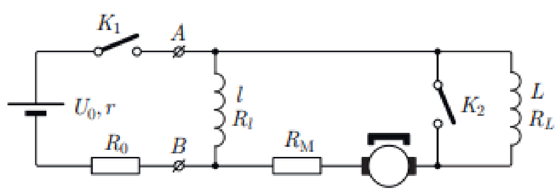

Figure 6: Problem 4

4. The inductance of the second coil L and its resistance R_L are such that the following equalities are true: $L = 10l$, $R_l = 5R_L$. It is known that the value of the current I_p lies between 10 A and 20 A. What is the current through the coil with inductance l at the moment key K_2 gets closed? Consider the numerical value of the parameter α to be known from part 3). *(2 points)*

Problem 5. Vacuum Gun (12 points)

In the recent years, a device called "Vacuum Gun" has drawn the attention of many enthusiasts who are engaged in scientific and engineering creativity. The polypropylene tube 2 (see the figure below) is tightly closed at one end by a foil plug 3(such that there is no air coming from this side of the tube), and the other end is open to the atmosphere. The difference in pressures accelerates the ping-pong ball 1. The inner diameter of the tube is about the same as the diameter of the ball. There's an inlet connection 4 near the plug, through which the tube is connected to a vacuum pump. So, the pressure to the right of the ball is always very small, meanwhile the pressure to the left is close to atmospheric pressure which equals P_0.

Turns out, if the length of the tube is big enough and the quality of pumping is good enough, it's possible to accelerate the ball to a speed which is enough for the ball to tear apart the foil plug and fly out of the tube. In a video available on internet, it's shown how a ball flies out of the tube and penetrates empty soda cans which are placed close to the end of the tube.

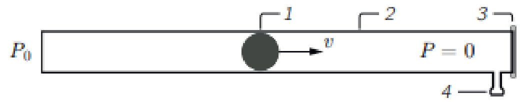

Figure 7: Problem 5

1. In the roughest model we assume that the pressure on the left side is $P_0 = 10^5$ Pa and on the right side it's zero. The difference in pressures is constant during the acceleration of the ball. There's no friction between the inner walls of the tube and the ball. To what maximum speed $v_{max,1}$ can a ball accelerate with mass $M = 2.7$ grams and diameter $d = 40$ mm in a tube with length $L = 2$ meters? *(1 point)*

   In a more accurate model we assume that air with mass $m(t)$ located on the left side to the ball also gets accelerated. More and more air from the atmosphere enters the tube and starts moving. It's also assumed that the left end of the tube has a size approximately about the same as the diameter of the tube. Outside the tube, the air has no speed. Inside the tube, the air moves with a speed equal to the speed of the ball and its density is ρ which is the same as the density of air outside. At the initial state, the position x and the speed of the ball are both equal to zero.

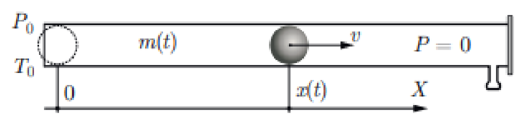

Figure 8: Problem 5

2. Find the more accurate maximum speed $v_{max,2}$ to which a ball with same dimensions as in part 1 can be accelerated in a tube the same length as in part 1. You can assume that the time of acceleration of the ball is the same as in part 1. The temperature of air is $T_0 = 293$ K and the molar mass μ is 29 g/mole. *(3 points)*

3. Assuming that only the temperature of air outside the tube $T_0 = 293$ K and molar mass $\mu = 29$ g/mole are known, find the maximum speed the ball can be accelerated to. The length of the tube is assumed to be large. *(3 points)*

4. The following parameters are given: M, S, P_0, ρ, μ. Find the formula of dependence of the coordinate $x(t)$ of the ball on time. *(4 points)*

   Note:. The following formula, $\Delta(x^2) = 2x\Delta x$, can be useful for small changes($\Delta x \ll x$) in x.

It's interesting that a group of students from Purdue University in USA succeeded in accelerating the ball to a speed v approximately equal to 420 m/s, $M = 1.23$ time bigger than the speed of sound by complicating the system a little, so that the ball for table tennis penetrated a table tennis racket. You can find the video by searching the phrase "Purdue Technology students build supersonic ping pong gun". In this enhanced system, there's a compressed air cylinder with a de Laval nozzle on the left side of the tube.