81st Moscow Olympiad for Students in Physics 2020
Grade 11, Round 2

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Problem 1. Oscillations Inside The Pipe (10 points)

A small body is located inside a horizontally oriented pipe of radius R, rotating with a certain angular velocity around its axis of symmetry. In the equilibrium position, the body is located below the pipe axis, at a distance of $0.8R$ vertically from it. Find the period of small oscillations of the body in a plane perpendicular to the pipe's axis.

Note: For small angle δ and any angle α, you can assume that $\sin(\alpha+\delta) = \sin\alpha + \delta\cos\alpha$, and $\cos(\alpha+\delta) = \cos\alpha - \delta\sin\alpha$.

Problem 2. Table With Thin Legs (12 points)

There is a table on a horizontal floor. The tabletop and the floor can be considered as absolutely solid; however, the legs are obeying Hooke's law for vertical deformations.

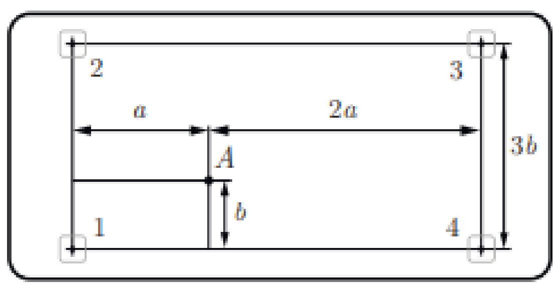


Figure 1: Problem 2

An object with mass 24 kg is placed on the table so that the center of the mass of the system is located at point A(see Figure 1, top view). By how much will the pressure on the table legs: $\Delta F_1, \Delta F_2, \Delta F_3$ and ΔF_4 change? The numbering of table legs is according to Figure 1.

Problem 3. Water-Steam-Water(14 points)

A certain amount of heat is given to water with mass $m = 180$ g, at constant pressure $p = 105$ Pa, so that all of the water becomes steam and its temperature rises up to $T_1 = 105°$C. Afterwards, the steam expands adiabatically and comes to saturation point, after which it condenses. The graph of the dependence of the pressure of saturated water vapor on temperature is shown in the figure below.

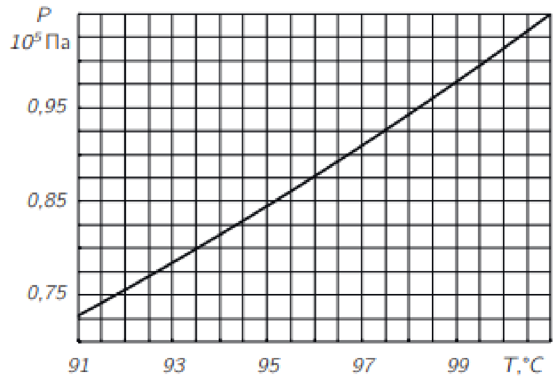


Figure 2: Problem 3

Considering the changes in steam parameters during adiabatic expansion to be small, determine the approximate temperature of water at the beginning of condensation.
What is the total work done by steam between heating (for evaporation) and cooling (for condensation)? The molar heat capacity of steam at constant volume is $C_V = 3R$.

Note: Note that for small changes in pressure and volume this approximation is valid: $\Delta(pV) = (\Delta p)V + p(\Delta V)$

Problem 4. Caustics (18 points)

A **Caustic** is defined as the envelope of rays that do not intersect at one point. In this case, the caustic is a curve that is tangent to all rays reflected from one surface or refracted at some interface. The light intensity near caustics increases, so the caustic curves are clearly visible to the naked eye and in photographs.

1. The gray curve in the figure is a caustic formed after a beam of parallel rays is reflected by a cylindrical surface M; the radius of surface being R. Determine the distance from axis of the cylinder, i.e. point O, to the vertex of the caustic, point A, namely distance OA, as well as the distance PA. OO_1 is the axis of symmetry and P_1P_2 is the tangent line to the caustic.*(4 points)*

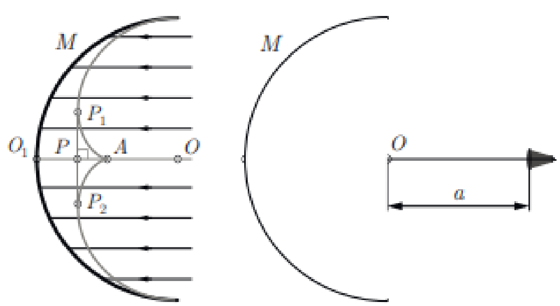
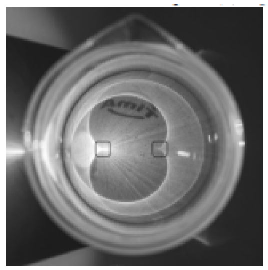


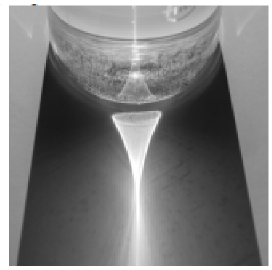
Figure 3: Problem 4

2. A point source illuminating a cylindrical surface of radius R is located at a distance $a = 4R$ from point O on the axis of symmetry of the OO_1 system. Determine the distance from point O to the vertex of the caustic formed by the reflected rays.*(4 points)*

   The glass with water in it is illuminated using a lantern. In each picture the light beam makes a different angle α with the horizontal surface of table as showin in Diagrams 1 and 2. In Diagram 1, the glass is photographed from above (the optical axis of the objective is perpendicular to the bottom of the glass), and it is illuminated from the right. In Diagram 2, the lens axis deviates slightly from the perpendicular.



Picture 1



Picture 2

3. This part is about the situation shown in picture 1. The angle α is about $45°$. Near the bottom of the glass, there is a curve with two peaks, which corresponds to the situation in the first part. In the figure, the peaks are marked using squares. Explain the observed pattern. *(3 points)*

4. This part is about situation shown in Diagram 2. The radius of the base of the glass is $R = 8$ cm. The lantern is located at a distance $a \approx 10R$ from the axis of the glass. The angle α can be considered small. Outside the glass, the caustic is visible, formed by the rays passing through the glass. Determine the approximate distance from the glass to the top of the caustic (to the top of the "cone"). The refractive index of water is $n \approx 1.33$. *(6 points)*

Problem 5. Thermal Inductor(18 points)

The Peltier element consists of two plates, separated by a large number of semiconductor blocks ,and it converts electrical energy into heat. It can also work in reverse mode – to generate EMF because of temperature difference between the plates. The second figure schematically shows a device based on a Peltier element. The bottom plate is in contact with

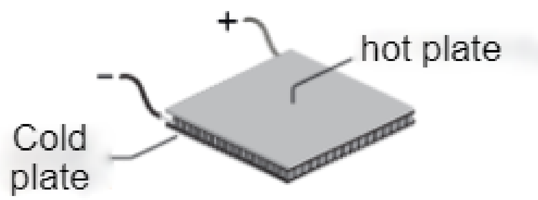


Figure 4: Problem 5

a heat reservoir, a large body whose temperature T_r can be considered constant. The upper plate is in contact with a body with heat capacity C and temperature T, which changes over time. A superconducting coil with inductance L is connected between ends of the element. Initially, $T = T_0$, $T_0 > T_r$.

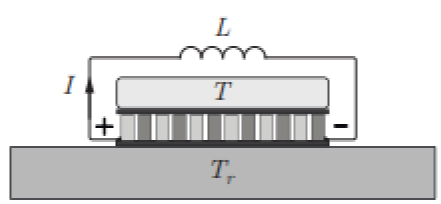


Figure 5: Problem 5

Under these conditions, the upper body gives the element during the time Δt the amount of heat $\Delta Q = \alpha T I \Delta t$, where α is a constant coefficient, I is the current through the element. The positive direction of current is the direction from "plus" to "minus". In this case, the Peltier element generates an EMF, which is equal to $\varepsilon = \alpha(T - T_r)$. The polarity of the generated EMF (for $T > T_r$) is shown in the figure. The resistance of the Peltier element is R.
It turns out in this system the periodic change in $T(t)$ and $I(t)$ is possible.In this problem we will investigate the fluctuations at various values of parameters: T_r, T_0, C, α, R which are considered to be known. the initial current in the circuit is zero: $I(0) = 0$.The temperature difference between the body and the reservoir can be considered small: $|T - Tr| \ll Tr$ at any time.

1. Let us consider an idealized, fantastic case, when the resistance R is equal to zero and there is no heat transfer between the heat reservoir and the body due to heat conduction.

   1a) Find the differential equation for $I(t)$. *(2 points)*

   1b) Transform the equation taking into account $|T - Tr| \ll Tr$ and find the frequency ω_0 of oscillations of the current. *(3 points)*

   1c) Find the dependence of temperature of the body on time: $T(t)$. *(1 point)*

2. Physically the case described in part 1 is not feasible. Consider the parameters of the system that are close to reality. Let the resistance of the element R be known, the power of heat transfer from the body to the reservoir is determined by the relation $P = k(T - T_r)$, where k is a known constant, heat is conducted uniformly between the plates. In this case oscillations will be damped.

   2a) Find the differential equation for $I(t)$. *(3 points)*

   2b) Show that the nonlinear terms are proportional to I^2 and $I\dot{I}$ (here dot shows the time derivative) in the differential equation from $2a$) can be neglected. Thus transform the equation into $\ddot{I} + 2\gamma\dot{I} + \omega^2 I = 0$ (equation for damped oscillations). Find the constants γ and ω in terms of given values. *(4 points)*

   2c) Assuming damping is weak ($\gamma^2 \ll \omega^2$), find the relative change in current amplitude $\dfrac{\Delta I_{max}}{I_{max}}$ over one time period. Express your answer in terms of γ and ω. *(3 points)*

3. The purpose of this device when it was first invented was to measure the heat capacity of bodies accurately. Parameters T_r, α, k, R, L are known, they have been measured before accurately. The experimenter has electrical measuring instruments, an oscilloscope and an alternating voltage generator, the frequency of which can be changed over a wide range. Describe in short a possible experimental design. *(2 points)*